PR1 - 2014 - 25 :

① $x[n] = 2\delta[n+1] + 2\delta[n+2]$

a) $x_p[n] \Rightarrow$ sinal par $\rightarrow$ por definição:

$$x_p[n] = \frac{x[n] + x[-n]}{2}$$

$$x[-n] = 2\delta[\underbrace{-n+1}_{n-1}] + 2\delta[\underbrace{-n+2}_{n-2}] = 2\delta[n-1] + 2\delta[n-2]$$

$$x_p[n] = \frac{\overbrace{2\delta[n+1] + 2\delta[n+2]}^{x[n]} + \overbrace{2\delta[n-1] + 2\delta[n-2]}^{x[-n]}}{2}$$

$$\boxed{x_p[n] = \delta[n+1] + \delta[n+2] + \delta[n-1] + \delta[n-2]}$$

$x_i[n] \Rightarrow$ sinal ímpar $\rightarrow$ por definição:

$$x_i[n] = \frac{x[n] + x[-n]}{2}$$

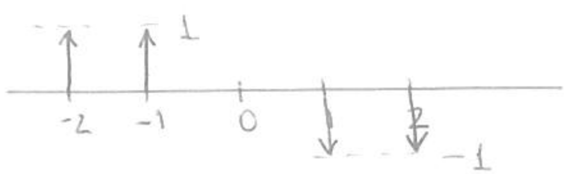
$$x_i[n] = \frac{\overbrace{2\delta[n+1] + 2\delta[n+2]}^{x[n]} - \overbrace{(2\delta[n-1] + 2\delta[n-2])}^{x[-n]}}{2}$$

$$\boxed{x_i[n] = \delta[n+1] + \delta[n+2] - \delta[n-1] - \delta[n-2]}$$

b) Esboçar $x_p[n]$ e $x_i[n]$

$x_p[n]$

$x_i[n]$



(2) $y[n] = \sum_{k=-2}^{n} k x[k+2]$

a) linear ou não linear?

Para ser linear o sistema deve obedecer ao princípio da superposição

$g\{\alpha x_1[n] + \beta x_2[n]\} = \alpha g\{x_1[n]\} + \beta g\{x_2[n]\}$

Substituindo no sistema que temos

$$g\{\alpha x_1(n) + \beta x_2(n)\} = \sum_{k=-2}^{n} k.(\alpha x_1[k+2] + \beta x_2[k+2])$$

$$= \sum_{k=-2}^{n} k\alpha x_1[k+2] + \sum_{k=-2}^{n} k\beta x_2[k+2] \quad (1)$$

$$\alpha g\{x_1[n]\} + \beta g\{x_2(n)\} = \alpha \sum_{k=-2}^{n} k x_1[k+2] + \beta \sum_{k=-2}^{n} k x_2[k+2] \quad (2)$$

Igualando (1) e (2)

$$\sum k\alpha x_1[k+2] + \sum k\beta x_2[k+2] = \alpha \sum k x_1[k+2] + \beta \sum k x_2[k+2]$$

como a igualdade é verdadeira, então o sistema é linear

b) causal ou não causal?

Para ser causal uma saída no instante n não deve depender da entrada de tempos futuros.

Por $y(n) = \sum_{k=-2}^{n} k x[k+2]$ nosso somatório vai de -2 até n e nossa entrada é $x[k+2]$ e a saída $y[n]$

Se $n = -2 \Rightarrow y[-2] = \sum_{-2}^{-2} k x(k+2)$

$y[-2] = (-2) \cdot x[0]$

$x[0]$ é futuro à $y[-2]$ ∴ Não causal

③ a) Função de transferência?

resposta ao impulso

$$h[n] = 2^{-n} u[n]$$

A transformada z da resposta ao impulso resulta na função de transferência.

$$h[n] \xrightarrow{z} H(z)$$

Propriedade $z\{a^n u[n]\} = \frac{z}{z-a} \quad |z| > |a|$

Fazendo

$$z\{2^{-n} u[n]\} = \frac{z}{z-a} = \frac{z}{z-1/2} \quad |z| > |1/2|$$

$a = 1/2$

$$\boxed{H(z) = \frac{z}{z-1/2} \quad |z| > |1/2|}$$

b) Solução forçada para $x[n] = 10$

$y_f = H(z) \cdot x[n]$ $\qquad x[n] = z^{+n} = 10 \cdot (1)^n$

$z = 1$

$$y_f = H(1) \cdot 10 = \frac{1}{1-1/2} 10 = 2 \cdot 10$$

$$\boxed{y_f = 20}$$

4) $X(z) = \dfrac{2z^2+5z}{(z+1)(z+2)} \quad |z|>2$

a) $x[0] = \lim\limits_{z\to\infty} X(z)$

$x(0) = \lim\limits_{z\to\infty} \dfrac{2z^2+5z}{(z^2+3z+2)}$

$\boxed{x[0]=2}$

b) $x[1] \Rightarrow$

$$\begin{array}{r|l} 2z^2+5z & z^2+3z+2 \\ \underline{2z^2+6z+4} & 2-z^{-1} \\ -z-4 & \uparrow \\ \underline{-z-3} & x[1] \\ -1 & \end{array}$$

$\boxed{x[1]=-1}$

c) $\sum\limits_{-\infty}^{+\infty} x[k] = \mathcal{Z}\{x[n]\}_{z=1}$

diverge

$\sum\limits_{-\infty}^{+\infty} x[k] = +\infty$, pois $z=1$
não pertence ao
domínio $z>2$.

5) $x(n) = \overbrace{(\rho^n u[n])}^{x_1[n]} * \overbrace{(\rho^n u[n])}^{x_2[n]}$

$X(z) = X_1(z)\cdot X_2(z)$

Propriedade $\mathcal{Z}\{a^n u(n)\} = \dfrac{z}{z-a} \quad |z|>|a|$

$X_1(z) = \dfrac{z}{z-\rho} = X_2(z)$

$X(z) = \dfrac{z^2}{(z-\rho)^2} \longrightarrow \boxed{x(n) = (n+1)\rho^n u[n]}$

6) $x(n) = n\,2^{-n}u[n] + u[-n-1]$

Transformada z e o domínio de existência

Propriedades ① $\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a} \quad |z|>|a|$

② $\mathcal{Z}\{-a^n u[-n-1]\} = \frac{z}{z-a} \quad |z|<|a|$ e ③ $\mathcal{Z}\{n^m x[n]\} = \left(-z\frac{d}{dz}\right)^m X(z)$

$$\underbrace{n\,2^{-n}u[n]}_{x[n]} \longrightarrow ③ \rightarrow \mathcal{Z}\{n\,x[n]\} = -z\frac{d}{dz}\left(\frac{z}{z-1/2}\right)$$

$\downarrow$ ①

$X(z) = \frac{z}{z-1/2}$

$$= -\frac{z((z-1/2)-z)}{(z-1/2)^2}$$

$$\boxed{+\frac{z/2}{(z-1/2)^2}} \qquad |z|>|1/2|$$

$$-(1)^n u[-n-1] \xrightarrow{②} -\frac{z}{z-1} \quad |z|<|1|$$

$$\boxed{X(z) = \frac{z/2}{(z-1/2)^2} - \frac{z}{z-1}} \qquad \boxed{1/2<|z|<1}$$

7) $X(z) = \frac{z}{(z-0{,}5)^2} \qquad |z|<0{,}5$

$$Y(z) = X(z^{-1}) = \frac{z^{-1}}{(z^{-1}-0{,}5)^2} = \frac{4z}{(z-2)^2} \qquad |z|>2$$

$y[n] = 4n(2)^{n-1}u[n] \longrightarrow x[n] = y[-n]$

$x(n) = -4n(2)^{-n-1}u[n]$

$$\boxed{x[n] = -n(2)^{-n+1}u[-n-1]}$$

(8)

$$\mathcal{E}\{z^{X}\} = \sum_k z^k \Pr\{X=k\} = \frac{26-5z}{z^2-12z+32} = \frac{26-5z}{(z-8)(z-4)} \quad |z|<4$$

a) $\Pr\{X=0\} = p[0] = \mathcal{E}\{z^X\}\Big|_{z=0} = \frac{26}{32} = \frac{13}{16}$

$$\boxed{\Pr\{X=0\} = \frac{13}{16}}$$

b) $\Pr\{X=1\} = p[1] = \frac{d\,\mathcal{E}\{z^X\}}{dz}\Big|_{z=0}$

$$= \frac{(-5)(\cancel{z^2}-12\cancel{z}+32)-(26-5\cancel{z})(2\cancel{z}-12)}{(\cancel{z^2}-12\cancel{z}+32)^2} = \frac{(-5)(32)-26.(-12)}{32^2}$$

$$\frac{(-5).(32)+26.12}{32^2} = \frac{-160+260+52}{32^2} = \frac{152}{32^2} = \frac{152}{1024}$$

$$\frac{76}{512} = \frac{38}{256} = \frac{19}{128} \qquad \boxed{\Pr\{X=1\} = \frac{19}{128}}$$

(9)

$$x[n] = 1 + 3\sin\left(\frac{2\pi}{3}n\right) + 2\cos\left(\frac{3\pi}{2}n\right)$$

a) $N = ?$ $\qquad 2\pi.\frac{1}{3} \qquad 2\pi.\frac{3}{4}$

$$3.n_1 = 4.n_2 = N = 12$$

$n_1 \to 4$, $n_2 \to 3$

$$\boxed{N = 12}$$

b) $c_k s = ?$

$$x(n) = e^{0} \cdot 1 + \frac{3}{2j}\left(e^{\frac{2\pi}{3}nj} - e^{\frac{-2\pi}{3}nj}\right) + \frac{2}{2}\left(e^{\frac{3\pi}{2}nj} + e^{\frac{-3\pi}{2}nj}\right)$$

$\frac{2\pi}{12} \cdot 4nj$ $\quad \frac{2\pi}{12}(-4nj)$ $\quad \frac{2\pi}{12} 9nj$ $\quad \frac{2\pi}{12}(-9nj)$

$$x[n] = \overset{k}{\hat{0}}{e}1 + \frac{3}{2j}\left(e^{\frac{2\pi}{12}\overset{k}{\hat{4}}nj} - e^{+\frac{2\pi}{12}\overset{k}{(-4)}nj}\right) + 1\left(e^{\frac{2\pi}{12}\overset{k}{(9)}nj} + e^{\frac{2\pi}{12}\overset{k}{(-9)}nj}\right)$$

$C_0 = 1 \quad C_4 = \frac{3}{2j} \quad C_{-4} = C_8 = -3/2j$

$C_9 = 1 \quad C_{-9} = C_3 = 1$

Demais nulos

(c) $P_m = \sum |C_k|^2 = \frac{1}{N} \sum |p[n]|$

Como no item anterior encontramos os $c_k s$ vamos usar que $P_m = \sum |c_k|^2$

$$P_m = \underbrace{1^2}_{C_0} + \underbrace{\left(\frac{3}{2}\right)^2}_{C_4} + \underbrace{\left(3/2\right)^2}_{C_8} + \underbrace{1^2}_{C_9} + \underbrace{1^2}_{C_3}$$

$$P_m = 3 + \frac{18}{4} = \frac{30}{4} = \frac{15}{2}$$

$$\boxed{P_m = \frac{15}{2}}$$

(10) $N=4 \qquad c_0=-j \quad c_1=j \quad c_2=1 \quad c_3=-1$

a) $\underbrace{x[0]}_{n}=?$ $\qquad x[n]=\sum c_k \exp\left(jk\frac{2\pi}{N}n\right)$

$\underline{\underline{n=0}}$

$$x[0]=\sum c_k = -j+j+1-1=0$$

$$\therefore \boxed{x[0]=0}$$

b) $\underbrace{x[1]}_{n=1}=?$

$$x[n]\to x[1]=\sum c_k e^{jk\frac{2\pi}{4}} \quad 4 \to N$$

$$\left.\begin{array}{l} c_0=-j \\ c_1=j \\ c_2=1 \\ c_3=-1 \end{array}\right\} \quad x[1]=-je^0+je^{j2\pi/4}+1e^{j2\cdot\frac{2\pi}{4}}-1e^{j3\frac{2\pi}{4}}$$

$$x[1]=-j+je^{j\pi/2}+e^{j\pi}-e^{j3\pi/2}$$

$$e^{j\pi/2}=\cancel{\cos\pi/2}^{\,0}+j\sin\pi/2=j$$

$$e^{j\pi}=\cos\pi+j\cancel{\sin\pi}^{\,0}=-1$$

$$e^{j3\pi/2}=\cancel{\cos(3\pi/2)}^{\,0}+j\sin(3\pi/2)=-j$$

$$x[1]=-j+\underbrace{j\cdot(j)}_{-1}-1-1\cdot(-j)$$

$$x[1]=-j-1-1+j \rightarrow \boxed{x[1]=-2}$$

c) $P_m=\sum|c_k|^2=\underbrace{1^2}_{c_0}+\underbrace{1^2}_{c_1}+\underbrace{1^2}_{c_2}+\underbrace{1^2}_{c_3}=4 \rightsquigarrow \boxed{P_m=4}$