RESOLUÇÃO PR2 – 2015

① $x(t) = G_2(t-1) + (-t-1)\,G_1(t+0{,}5)$, determine e esboce $x(-t-2)$

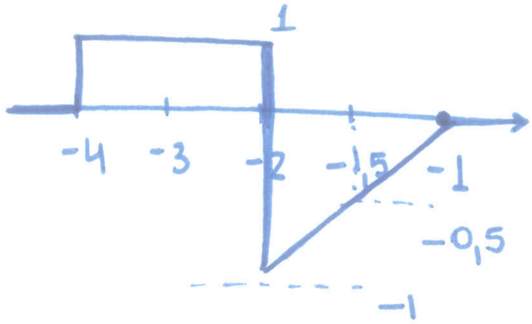
$\downarrow$

$(-t-2)$ $\rightsquigarrow$ $x(-t-2) = G_2(-t-2-1) + (-(-t-2)-1)\,G_1(-t-2+0{,}5)$

$$x(-t-2) = G_2(-t-3) + (t+1)\,G_1(-t-1{,}5)$$

$$\boxed{x(-t-2) = G_2(t+3) + (t+1)\,G_1(t+1{,}5)}$$

$\Downarrow$



② $\dot{y}(t) + 6t\,y(t) = \ddot{x}(t) + 3\dot{x}(t) + 5x(t)$

linear = ?
invariante = ?
causal = ?

- Linear pq vale o princípio da superposição

→ Variante pq o coef. de $6t\,y(t)$ → depende de "t".

→ não - causal pq a ordem do numerador de $h(t)$ é maior que do denominador

$$y(p+6t) = x(t)(p^2+3p+5)$$

$$h(t) = \frac{p^2+3p+5}{p+6t} \quad 2 > 1$$

③ $y_f(t) = ? \longrightarrow x(t) = e^t \longrightarrow y(t) = H(s) \cdot e^{st}$

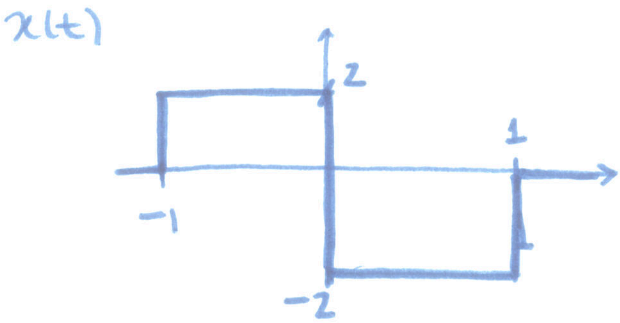
$$\dot{y} + 2y = x$$

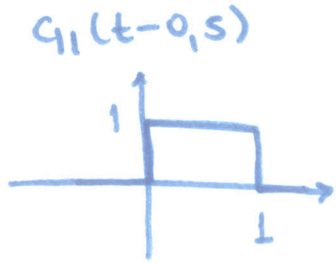
$$sH(s) \cdot e^t + 2H(s)e^t = e^t \longrightarrow H(s) = \frac{1}{s+2}$$

$$y_f(t) \rightarrow x(t) = e^t = e^{\overset{=s}{1} \cdot t}$$

$$y_f(t) = e^t H(1) = e^t \cdot \frac{1}{3} \Rightarrow \boxed{y_f(t) = \frac{e^t}{3}}$$

④ Determine e esboce a convolução $x(t) = 2G_1(t+0{,}5) - 2G_1(t-0{,}5)$ com $G_1(t-0{,}5)$





$$x(t) * G_1(t-0{,}5) = x(t) * (u(t) - u(t-1)) =$$

$$= \underbrace{x(t) * u(t)}_{I_x(t)} - \underbrace{x(t) * u(t-1)}_{I_x(t-1)}$$

$$I_x(t) = \int_{-1}^{t} 2\,dt \cdot G_1(t+0{,}5) + \left(\int_0^t -2\,dt + 2\right) G_1(t-0{,}5)$$

$$I_x(t) = (2t+2)G_1(t+0{,}5) + (-2t+2)G_1(t-0{,}5)$$

$$x(t) * G_1(t-0{,}5) = \overbrace{(2t+2)\,G_1(t+0{,}5) + (-2t+2)\,G_1(t-0{,}5)}^{I_x(t)} -$$

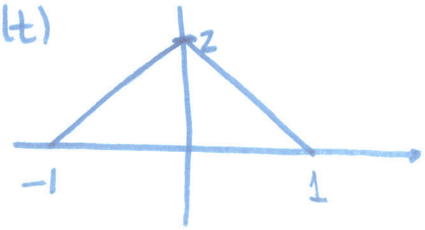
$$- \underbrace{\Big((2(t-1)+2)\,G_1(t-0{,}5) + (-2(t-1)+2)\,G_1(t-1{,}5) \Big)}_{I_x(t-1)}$$

$$x(t) * G_1(t-0{,}5) = (2t+2)\,G_1(t+0{,}5) + (-2t+2)\,G_1(t-0{,}5) +$$

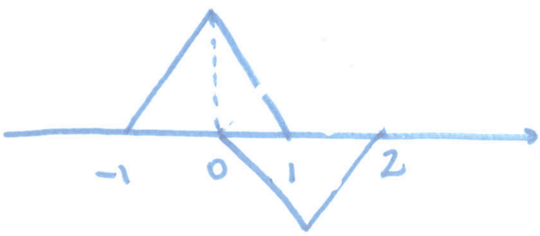
$$+ (-2t)\,G_1(t-0{,}5) + (+2t-4)\,G_1(t-1{,}5)$$

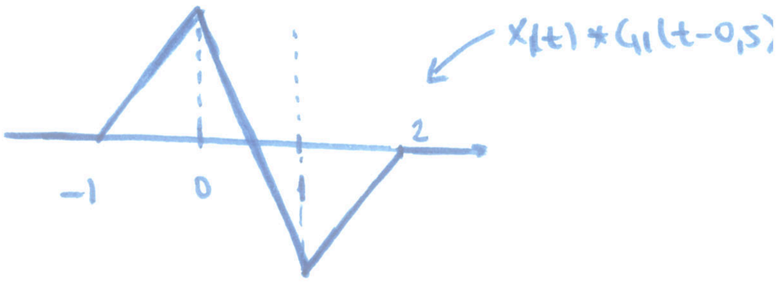
$$\rightarrow x(t) * G_1(t-0{,}5) = (2t+2)\,G_1(t+0{,}5) + (-4t+2)\,G_1(t-0{,}5) + (2t-4)\,G_1(t-1{,}5)$$

$I_x(t)$



$x(t) * G_1(t-0{,}5)$





5 a) $h(t) = ?$

$$y(t) = \int_{t-2}^{t+2} (t-\beta)\,x(\beta)\,d\beta$$

$$h(t) = \mathcal{Y}\{\delta(t)\} = \int_{t-2}^{t+2} (t-\beta)\,\delta(\beta)\,d\beta$$

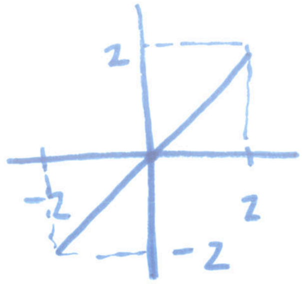
$$h(t) = (u(t+2) - u(t-2))\,t = G_4(t).t$$

$$\boxed{h(t) = t \,.\, G_4(t)}$$

b) causal? BIBO?

causal $\rightarrow h(t) = 0$ p/ $t < 0$

$h(t) =$



para $\left.\begin{array}{l} t=-1 \\ t=-2 \end{array}\right\} h(t) \neq 0$

$\therefore$

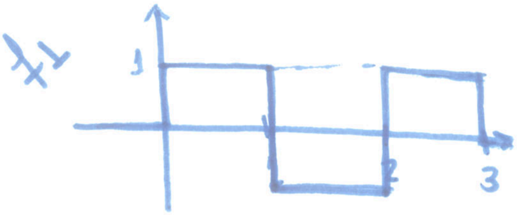
NÃO CAUSAL

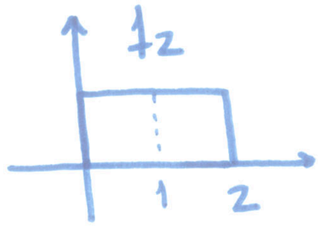
BIBO

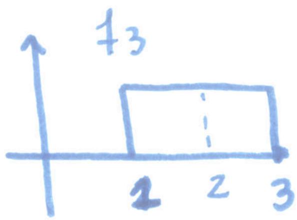
$$\int |h(t)|\,dt < \infty = \int_{-2}^{2} t\,dt = \frac{t^2}{2} = 0 \therefore$$ **BIBO-ESTÁVEL**

⑥ $f_1(t) = G_1(t-0{,}5) - G_1(t-1{,}5) + G_1(t-2{,}5)$

$f_2(t) = G_2(t-1) \quad f_3 = G_2(t-2)$







$\boxed{g_1 = f_1}$

Gram-Schmidt:

$$g_2 = f_2 - \frac{\langle g_1 f_2 \rangle}{\langle g_1^2 \rangle} g_1$$

$$\langle g_1 f_2 \rangle = \int_0^1 dt + \int_1^2 -1 \cdot 1\,dt + \int_2^3 0\,dt$$

$$1 - 2 + 1 = 0$$

$$\langle g_1^2 \rangle = \int_0^1 1\,dt + \int_1^2 1\,dt + \int_2^3 1$$

$$\cancel{1} + \cancel{2} - \cancel{1} + 3 - \cancel{2} = 3$$

$$g_2 = f_2 - 0 \rightarrow \boxed{g_2 = f_2}$$

$$g_3 = f_3 - \frac{\cancel{\langle g_1 f_3\rangle}^{\,0}}{\langle g_1^2\rangle} g_1 - \frac{\cancel{\langle g_2 f_3\rangle}^{\,1}}{\cancel{\langle g_2^2\rangle}_{\,2}} g_2$$

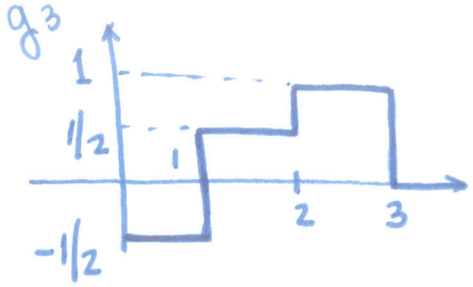
$$\langle g_1 f_3\rangle = \cancel{\int_0^1 0\,dt} + \int_1^2 -1\,dt + \int_2^3 1\,dt$$

$$\boxed{g_3 = f_3 - \frac{1}{2} g_2}$$

$$\langle g_1 f_3\rangle = 0$$

$$\langle g_2 f_3\rangle = \int_1^2 1\,dt = 1$$

$$\langle g_2^2\rangle = \int_0^2 1\,dt = 2$$



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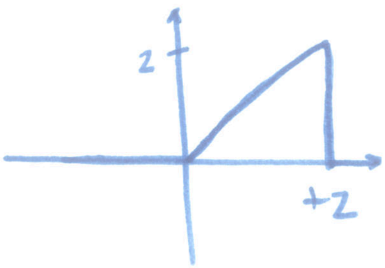
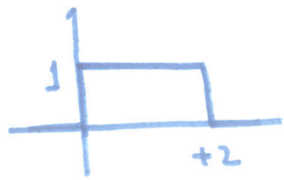
$$\epsilon(t) = \underbrace{(2t - t^2)\,G_2(t-1)}_{y(t)} - \Big(a\,\underbrace{t\,G_2(t-1)}_{x_1(t)} + b\,\underbrace{G_2(t-1)}_{x_2(t)}\Big)$$

$$\begin{bmatrix} \langle x_1^2\rangle & \langle x_1 x_2\rangle \\ \langle x_1 x_2\rangle & \langle x_2^2\rangle \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \langle y x_1\rangle \\ \langle y x_2\rangle \end{bmatrix}$$

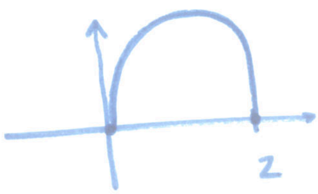
$x_1(t)$



$x_2(t)$



$y(t)$



$$\langle x_2^2\rangle = \int_0^2 dt = 2$$

$$\langle y x_1\rangle = \int_0^2 (2t^2 - t^3)\,dt$$

$$\frac{2t^3}{3} - \frac{t^4}{4}\Big|_0^2 = \frac{16}{3} - \frac{16}{4}$$

$$\langle x_1^2\rangle = \int_0^2 t^2\,dt = \frac{t^3}{3}\Big|_0^2 = 8/3$$

$$\frac{64-48}{12} = \frac{16}{12}$$

$$\langle x_1 x_2\rangle = \int_0^2 t\,dt = \frac{t^2}{2}\Big|_0^2 = 2$$

$$\langle y x_1\rangle = 4/3$$

$$\langle y x_2 \rangle = \int_0^2 (2t - t^2)\,dt = t^2 - \frac{t^3}{3}\Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

$$\begin{bmatrix} 8/3 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

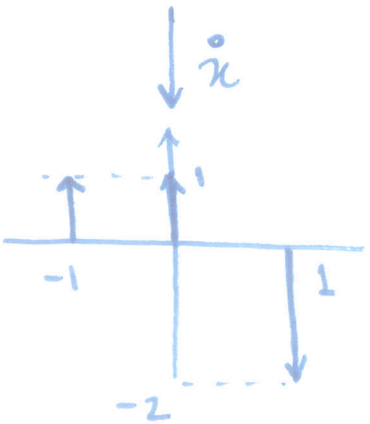
$$\begin{cases} 8/3\, a + 2b = 4/3 \\ 2a + 2b = 4/3 \end{cases} \qquad 8/3\, a - 2a = 0$$

$$a\,\frac{8-6}{3} = 0$$

$$\boxed{b = \frac{2}{3}} \qquad \boxed{a = 0}$$

(8) a) $c_k = ?$

$$x(t) = \sum p(t - k10), \quad p(t) = u(t+1) + u(t) - 2u(t-1)$$



$$c_k = \frac{1}{10 j \cdot \frac{2\pi k}{10}} \left(e^{j \cdot \frac{2\pi}{10} k} + 1 - 2 e^{-j\frac{2\pi}{10} k} \right)$$

$$c_k = \frac{1}{10} \left(\frac{e^{j\frac{\pi}{5}k} + 1 - 2e^{-j\frac{\pi}{5}k}}{j\pi k/5} \right)$$

b) $c_0 = \frac{\text{Área}}{T} = \frac{\text{Área}}{10} = \frac{1+2}{10} = \boxed{\frac{3}{10}}$

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$$x(t) = 3j + j\cos(4t) - 3\cos(6t)$$

a) $T = ?$

$$T_1 \rightarrow \frac{2\pi}{T_1} = 4 \rightarrow T_1 = \frac{2\pi}{4}$$

$$T_2 \rightarrow \frac{2\pi}{T_2} = 6 \rightarrow T_2 = \frac{2\pi}{6}$$

$$T = pT_1 = qT_2 \Rightarrow T = \frac{\cancel{2\pi}}{\cancel{4}_2} \cdot p = \frac{\cancel{2\pi}}{\cancel{6}_3} q$$

$$T = \frac{\overset{=2}{p}}{2}\pi = \frac{\overset{=3}{q}}{3}\pi$$

$$T = \pi \rightarrow \omega_0 = 2$$

b)

$$\frac{2\pi}{\pi} \cdot 2 = 4 \qquad \frac{2\pi}{\pi} \cdot 3 = 6$$

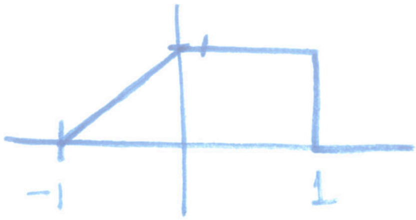
$$x(t) = 3j + j/2\left(e^{j\frac{2\pi}{\pi}\overset{k}{2}t} + e^{\overset{k}{-j\frac{\pi 2}{\pi}2t}}\right) - \frac{3}{2}\left(e^{j\frac{2\pi}{\pi}\overset{k}{.3}t} + e^{\overset{k}{-j\frac{2\pi}{\pi}3t}}\right)$$

$$\boxed{c_0 = 3j} \quad \boxed{c_2 = c_{-2} = j/2} \quad \boxed{c_3 = c_{-3} = -\frac{3}{2}}$$

Demais Nulos

c) $P_m = \frac{1}{N}\sum |a_k|^2 = 9 + \frac{1}{4}\cdot 2 + \frac{9}{4}\cdot 2 = 9 + \frac{1}{2} + \frac{9}{2} = \boxed{14}$

10 $P_m = ?$ $\quad x(t) = \sum_{k=-\infty}^{\infty} p(t-k10) \quad p(t) = (t+1)\,G_1(t+0{,}5) + G_1(t-0{,}5)$



$$P_m = \frac{1}{T}\int_T |x(t)|^2\,dt$$

$$P_m = \frac{1}{10}\left(\int_{-1}^{0}(t+1)^2\,dt + \int_0^1 1^2\,dt\right) = \frac{1}{10}\left(\frac{t^3}{3}+t^2+t\Big|_{-1}^{0} + t\Big|_0^1\right)$$

$$P_m = \frac{1}{10}\cdot\left(-\left(-1/3 + \cancel{1} - \cancel{1}\right) + 1\right) = \frac{1}{10}\left(1/3+1\right)$$

$$P_m = \frac{\cancel{4}}{3}\cdot\frac{1}{\cancel{10}} = \boxed{\frac{2}{15}}$$