RESOLUÇÃO PR2-2016

1. $x(t) = G_2(t+1) + (t/2 - 1)\, G_2(t-1)$, determine e esboce $x(-2t+2)$

$\downarrow$

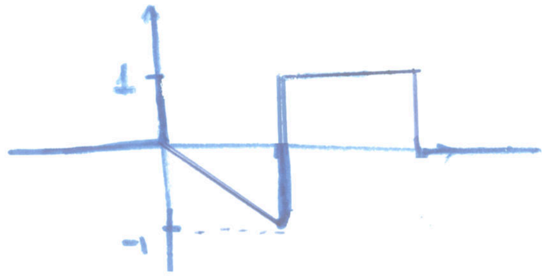
$-2t+2$

↳ contração de 2

$$x(-2t+2) = G_1(-2t+2+1) + \left(\frac{-2t+2}{2} - 1\right) G_1(-2t+2-1)$$

$$x(-2t+2) = G_1(-2t+3) + (-t)\, G_1(-2t+1)$$

$$\boxed{x(-2t+2) = G_1(t-1{,}5) - t \cdot G_1(t-0{,}5)}$$



2. $y(t) = \cos(2t + \pi/6)\, x(t)$

linear?
causal?
invariante?

linear vale o princípio da superposição

causal só depende de $x(t)$

invariante no tempo: $y_2(t) = y_1(t-a)$ $\quad x_2(t) = x_1(t-a)$

$$y_2(t) = \cos(2t + \pi/6)\, x_2(t)$$

$$y_1(t-a) = \cos(2(t-a) + \pi/6)\, x_1(t-a) \neq x_1(t-a)$$

3 $y_f(t) \to p/\ x(t) = \cos^2(3t)$

$$\ddot{y} + 40y = x \qquad x(t) = e^{st}$$

$$y(t) = H(s)e^{st}$$

$$s^2 H(s) e^{st} + 40 e^{st} H(s) = e^{st}$$

$$H(s) = \frac{1}{s^2+40}$$

$$x(t) = \left(\frac{e^{+3jt} + e^{-3jt}}{2}\right)^2 = \frac{e^{+6jt}}{4} + \frac{1}{2} + \frac{e^{-6jt}}{4}$$

$$x(t) = \frac{1}{4}e^{6jt} + \frac{1}{2}e^{0t} + \frac{e^{-6jt}}{4}$$

$$y_f(t) = \frac{1}{4}e^{6jt} H(6j) + \frac{1}{2}H(0) + \frac{1}{4}e^{-6jt} H(-6j)$$

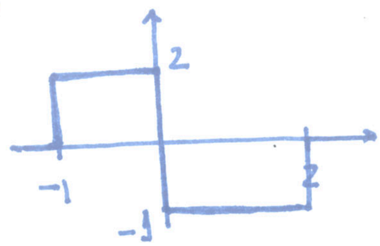
$$y_f(t) = \frac{1}{4}e^{6jt} \frac{1}{-36+40} + \frac{1}{2 \cdot 40} + \frac{1}{4}e^{-6jt} \frac{1}{-36+40}$$

$$\boxed{y_f(t) = \frac{1}{80} + \frac{1}{8}\cos(6t)}$$

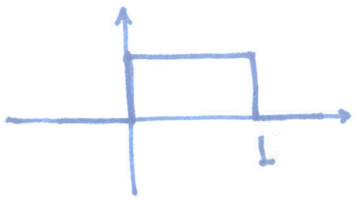
4. Determine e esboce a convolução

$$x(t) = 2G_1(t+0{,}5) - G_2(t-1) \text{ com } G_1(t-0{,}5)$$

$x(t)$



$G_1(t-0{,}5)$



$$x(t) * G_1(t-0{,}5) = x(t) * (u(t) - u(t-1)) =$$

$$= \underbrace{x(t) * u(t)}_{I_x(t)} - \underbrace{x(t) * u(t-1)}_{I_x(t-1)}$$

$$I_x(t) = \int_{-1}^{t} 2\,dt \cdot G_1(t+0{,}5) + \left(\int_0^t -dt + \int_{-1}^{0} 2\,dt\right) G_2(t-1)$$

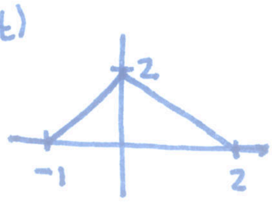
$$I_x(t) = (2t+2)\,G_1(t+0{,}5) + (-t+2)\,G_2(t-1)$$

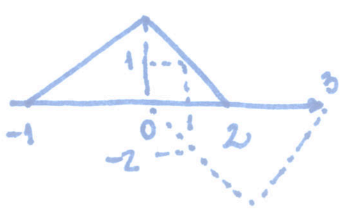
$$x(t) * G_1(t-0{,}5) = \overbrace{(2t+2)\,G_1(t+0{,}5) + (-t+2)\,G_2(t-1)}^{I_x(t)} -$$

$$\underbrace{-(2(t-1)+2)\,G_1(t-0{,}5) + (-(t-1)+2)\,G_2(t-2)}_{I_x(t-1)}$$

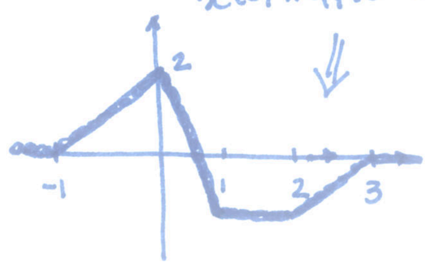
$$x(t) * G_1(t-0{,}5) = (2t+2)\,G_1(t+1/2) + (-t+2)\,G_2(t-1) + (-2t)\,G_1(t-1/2) + (t-3)\,G_2(t-2)$$

$I_x(t)$





$x(t) * G_1(t-0{,}5)$



outra forma de escrever $x(t) * G_1(t-0{,}5)$

$$(2t+2)\,G_1(t+0{,}5) + (-3t+2)\,G_1(t-0{,}5) - G_1(t-1{,}5) + (t-3)\,G_1(t-2{,}5)$$

5) a) $h(t) = ?$

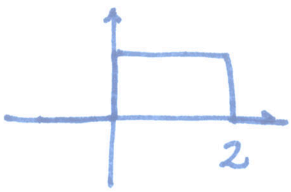
$$y(t) = \int_{t-2}^{t} x(\beta)\exp(t-\beta)\,d\beta$$

$$h(t) = \int_{t-2}^{t} \delta(\beta)\exp(t-\beta)\,d\beta = e^{t}(\mu(t) - \mu(t-2))$$

$$\boxed{h(t) = e^{t}(\mu(t) - \mu(t-2))}$$

b) causal? BIBO?

causal $\rightsquigarrow$ $h(t) = 0$ p/ $t < \emptyset$



$h(t) = \emptyset$ p/ $t < 0$

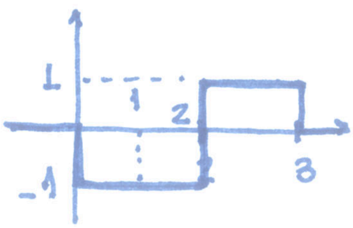
$\therefore$ <u>causal</u>

BIBO $\rightsquigarrow$ $\int |h(t)|\,dt < \infty$ $\quad \int_0^2 e^{t}\,dt = e^{2} - e^{0} = e^{2} - 1 < \infty$

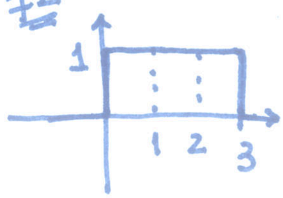
<u>BIBO ESTÁVEL</u>

6) $f_1(t) = -G_2(t-1) + G_1(t-2{,}5)$ $\quad f_2(t) = G_3(t-1{,}5)$ $\quad f_3(t) = G_2(t-2)$

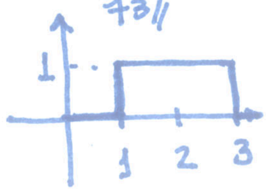
$f_1 =$



f_2



f_3



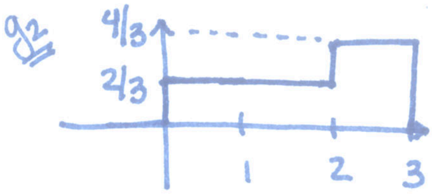
<u>Gram schmidt</u>

$\boxed{g_1 = f_1}$

$$g_2 = f_2 - \frac{\langle g_1 f_2\rangle}{\langle g_1^2\rangle} g_1$$

$$\langle g_1 f_2\rangle = \int_0^2 -1.1\,dt + \int_2^3 1\,dt = -2+3-2 = \boxed{-1}$$

$$\langle g_1{}^2\rangle = \int_0^3 1\,dt = \boxed{3} \qquad g_2 = f_2 + \frac{1}{3} g_1$$



$$\boxed{g_2 = \frac{2}{3} G_2(t-1) + \frac{4}{3} G_1(t-2{,}5)}$$

$$g_3 = f_3 - \frac{\langle g_1, f_3\rangle}{\langle g_1{}^2\rangle} g_1 - \frac{\langle g_2 f_3\rangle}{\langle g_2{}^2\rangle} g_2$$

$$\langle g_1 f_3\rangle = \int_1^2 -1.1.dt + \int_2^3 1.dt = -2+1+3-2 = \emptyset$$

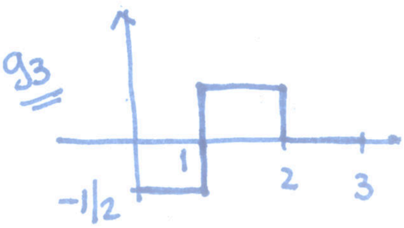
$$\langle g_2 f_3\rangle = \int_1^2 \frac{2}{3}.1\,dt + \int_2^3 1.4/3 = \frac{4}{3} - \frac{2}{3} + 4 - \frac{8}{3}$$

$$4 - \frac{6}{3} = \boxed{2}$$

$$\langle g_2{}^2\rangle = \int_0^2 4/9\,dt + \int_2^3 16/9\,dt = \frac{8}{9} + \frac{16}{3} - \frac{32}{9}$$

$$\frac{16}{3} - \frac{24}{9} = \frac{}{9} = \frac{24}{9} = \boxed{\frac{8}{3}}$$

$$g_3 = f_3 - \frac{2}{8}.3 g_2 = f_3 - \frac{3}{4} g_2$$

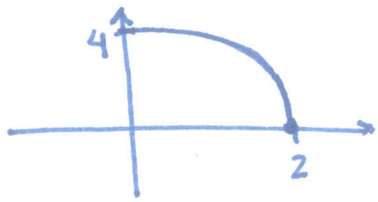


$$\boxed{g_3 = -1/2\, G_1(t-0{,}5) + 1/2\, G_1(t-1{,}5)}$$

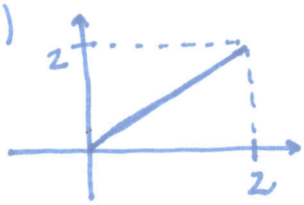
(7) $\epsilon(t) = \underbrace{(-t^2+4)\, G_2(t-1)}_{y(t)} - \left(a\, \underbrace{t\, G_2(t-1)}_{x_1(t)} + b\, \underbrace{G_2(t-1)}_{x_2(t)}\right)$

$$\begin{bmatrix} \langle x_1^2 \rangle & \langle x_1 x_2 \rangle \\ \langle x_1 x_2 \rangle & \langle x_2^2 \rangle \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \langle y x_1 \rangle \\ \langle y x_2 \rangle \end{bmatrix}$$

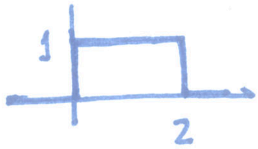
$y(t)$



$x_1(t)$



$x_2(t)$



$\langle x_1^2 \rangle = \int_0^2 t^2\, dt = \boxed{\frac{8}{3}}$

$\langle x_1 x_2 \rangle = \int_0^2 t\, dt = \frac{t^2}{2}\Big|_0^2 = \boxed{2}$

$\langle x_2^2 \rangle = \int_0^2 1.dt = \boxed{2}$

$\langle y x_1 \rangle = \int_0^2 (-t^3+4t)\, dt =$

$-\frac{t^4}{4} + 2t^2 \Big|_0^2 = -4+8 = \boxed{4}$

$\langle y x_2 \rangle = \int_0^2 (-t^2+4)\, dt =$

$-\frac{t^3}{3} + 4t \Big|_0^2 = -\frac{8}{3} + 8 = \boxed{\frac{16}{3}}$

$$\begin{bmatrix} 8/3 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 16/3 \end{bmatrix} \qquad \begin{cases} 8/3\, a + 2b = 4 \\ 2a + 2b = 16/3 \end{cases}$$

$(8/3 - 2)\, a = 4 - \frac{16}{3}$

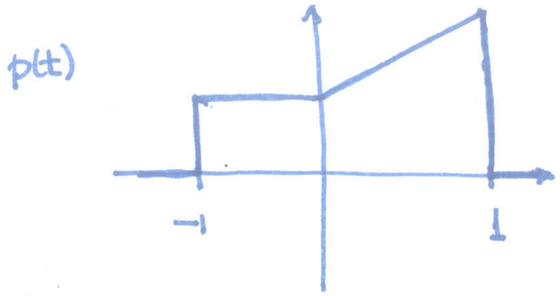
$\frac{2}{3} a = -\frac{4}{3} \rightarrow \boxed{a = -2}$

$2.(-2) + 2b = 16/3 \rightarrow 2b = \frac{16}{3} + 4 = \frac{28}{3}$

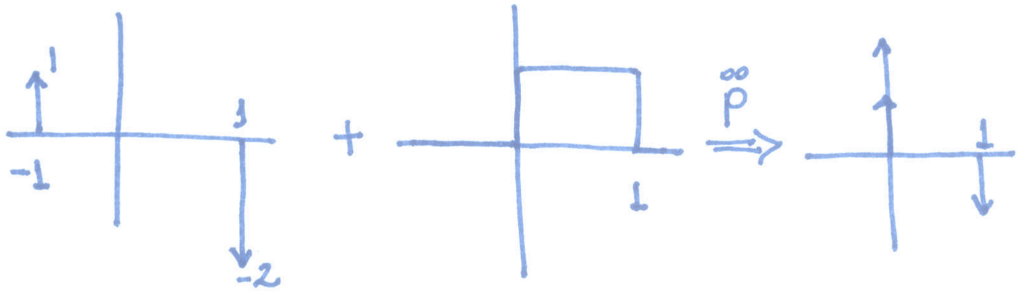
$\boxed{b = \frac{14}{3}}$

8 a) $c_k = ?$

$$x(t) = \sum p(t-4k) \qquad p(t) = G_1(t+0{,}5) + (t+1)\,G_1(t-0{,}5)$$



$\Downarrow \dot{p}(t)$



$$c_k = \frac{1}{4}\left(\frac{e^{\frac{2\pi}{4}jk} - 2e^{-\frac{2\pi}{4}jk}}{\frac{2\pi}{4}kj} + \frac{1 - e^{-2\pi/4\,jk}}{\left(\frac{2\pi}{4}kj\right)^2}\right)$$

$$c_k = \frac{1}{\cancel{4}}\left(\cancel{4}\cdot\left(\frac{e^{\pi/2\,jk} - 2e^{-\pi/2\,jk}}{2\pi kj}\right) + \cancel{4}\,\frac{\left(e^{-2\pi/2\,jk} - 1\right)}{\pi^2 k^2}\right)$$

$$\boxed{c_k = \left[\frac{e^{\pi/2\,jk} - 2e^{-\pi/2\,jk}}{2\pi kj} + \frac{e^{-\pi/2\,jk} - 1}{\pi^2 k^2}\right]}$$

b) $c_0 = \frac{\text{Área}}{T} = \frac{\text{Área}}{4} = \frac{1+1+1/2}{4} = \boxed{\frac{5}{8}}$

9) $x(t) = 5\cos^2(2t+\pi/6) - 2j\exp(j3t)$

$$5\left(\frac{e^{j(2t+\pi/6)}}{2} + \frac{e^{-j(2t+\pi/6)}}{2}\right)^2 - 2je^{j3t}$$

$$5\left(\frac{e^{(4t+\pi/3)j}}{4} + \frac{1}{2} + \frac{e^{-(4t+\pi/3)j}}{4}\right) - 2je^{j3t}$$

$$T_1 \rightarrow \frac{2\pi}{T_1} = 4 \rightarrow T_1 = \frac{\pi}{2}$$

$$T_2 \rightarrow \frac{2\pi}{T_2} = 3 \rightarrow T_2 = \frac{2\pi}{3}$$

$$T = \frac{2\pi}{3}\cdot \underset{3}{p} = \frac{\pi}{2}\cdot \underset{4}{q} = 2\pi \rightarrow \boxed{\omega_0 = 1}$$

$$\boxed{T = 2\pi}$$

$$\frac{2\pi}{2\pi}\cdot 4 = 4 \qquad \frac{2\pi}{2\pi}\cdot 3 = 3$$

b) $x(t) = \frac{5}{2} + \frac{5}{4}\left(e^{4tj}\cdot e^{\pi/3j} + e^{-4tj}e^{-\pi/3j}\right) - 2je^{j3t}$

$$\boxed{c_0 = 5/2} \quad \boxed{c_4 = \frac{5}{4}e^{\pi/3j}} \quad \boxed{c_{-4} = \frac{5}{4}e^{-\pi/3j}}$$

$$\boxed{c_3 = -2j}$$

Demais Nulos

c) $P_m = \sum|c_k|^2 = \frac{25}{4} + \frac{25}{16}\cdot 2 + 4 = \frac{50}{8} + \frac{25}{8} + \frac{32}{8} = \boxed{\frac{107}{8}}$

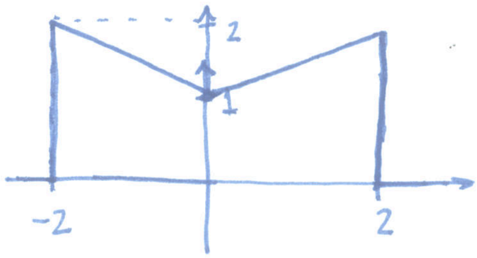
10 $x(t) = \sum p(t-k5) \quad p(t) = (-t+1)G_1(t+0{,}5) + (t+1)G_1(t-0{,}5)$

a) $T = ? \rightarrow y(t) = x(t/2)$

$\boxed{T = 10}$

$x(t/2) = (-t/2+1)G_2(t/2+0{,}5) + (t/2+1)G_2(t/2-0{,}5)$

$x(t/2) = (-t/2+1)G_2(t+1) + (t/2+1)G_2(t-1)$



$P_m = \frac{1}{10} \cdot 2 \int_0^2 (t/2+1)^2 dt = \frac{1}{5}\left(\frac{t^3}{12} + \frac{t^2}{2} + t\right)\Big|_0^2$

$\frac{1}{5}\left(\frac{\cancel{8}^{2}}{\cancel{12}_{3}} + 2 + 2\right) = \frac{1}{5}\left(4 + \frac{2}{3}\right)$

$\boxed{P_m = \frac{14}{15}}$