Resolução PR2-2017:

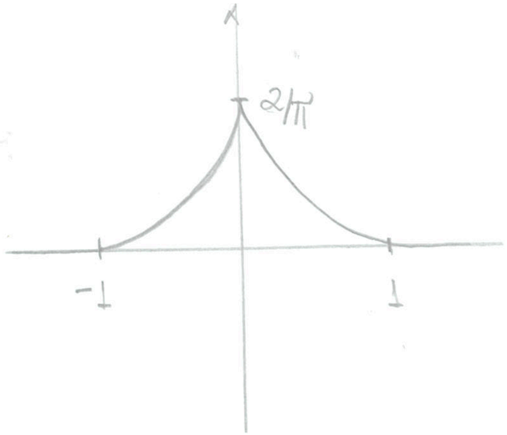
1) $y(t) = x(t) * u(t) = I_x(t)$

$$x(t) = \cos\left(\pi/_2 t\right) G_1(t+0{,}5) - \cos\left(\pi/_2 t\right) G_1(t-0{,}5)$$

$$I_x(t) = G_1(t+0{,}5) \cdot \int_{-1}^{t} \cos(\pi/_2 t)\,dt + G_1(t-0{,}5)\left[\int_{-1}^{0} \cos(\pi/2t)\,dt + \int_{0}^{t} -\cos(\pi/_2 t)\,dt\right]$$

$$G_1(t+0{,}5)\left[\frac{2}{\pi}\,\mathrm{sen}(\pi/2t)\Big|_{-1}^{t}\right] + G_1(t-0{,}5)\left[\frac{2}{\pi} + \left(-\frac{2\,\mathrm{sen}(\pi/2t)}{\pi}\Big|_{0}^{t}\right)\right]$$

$$\boxed{I_x(t) = \frac{2}{\pi} G_1(t+0{,}5)\left(\mathrm{sen}(\pi/2t)+1\right) + \frac{2}{\pi} G_1(t-0{,}5)\left(1 - \mathrm{sen}(\pi/2t)\right)}$$



2) $y(t) = \int_{-\infty}^{t+2} x(\beta - 2)\,d\beta$

linear? $g\{a x_1(t) + b x_2(t)\} = g\{a x_1(t)\} + g\{b x_2(t)\}$

linear

causal? A integral vai de $x(-\infty) \to x(t)$ $\therefore$ causal

BIBO? $\int_{-\infty}^{t+2} x(\beta-2)\,d\beta$ p/ uma entrada limitada e a integral indo de $-\infty \rightarrow t+2$ o sistema é BIBO //

Invariante?

$$y(t+a) = g\{x(t+a)\}$$

$$y(t+a) = \int_{-\infty}^{t+a+2} x(\beta-2)\,d\beta \xrightarrow{\alpha=\beta-2} \int_{-\infty}^{t+a} x(\alpha)\,d\alpha$$

|| Invariante

$$g\{x(t+a)\} = \int_{-\infty}^{t+2} x(\beta+a-2)\,d\beta \xrightarrow{\alpha=\beta+a-2} \int_{-\infty}^{t+a} x(\alpha)\,d\alpha$$

3 $H(s) = ?$

a) $\ddot{y} + 4y = x$ $\quad x = e^{st}$

$$\begin{cases} y = H(s)\,e^{st} \\ \dot{y} = s\,e^{st}H(s) \\ \ddot{y} = s^2 e^{st} H(s) \end{cases}$$

substituindo

$$s^2 e^{st} H(s) + 4e^{st} H(s) = e^{st}$$

$$H(s) = \frac{e^{st}}{e^{st}(s^2+4)} \longrightarrow \boxed{H(s) = \frac{1}{s^2+4}}$$

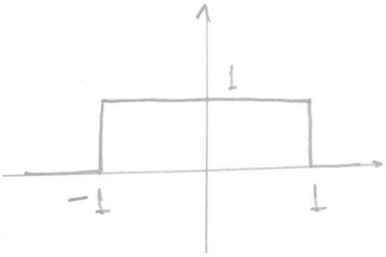
b) $y_f(t)$ para $x(t) = 4 - 4e^{4jt} = 4e^{0t} - 4e^{4jt}$ $\quad s$

$$y_f(t) = x(t)\,H(s) \longrightarrow y_f(t) = H(0)\cdot 4 - 4e^{4jt}H(4)$$

$$\boxed{y_f(t) = 1 + e^{4jt}/3}$$

(4) $G_2(t) * y(t)$ $\quad y(t) = x(2-t)$ $\quad x(t) = G_2(t-3)$

$$y(t) = G_2(2-t-3) = G_2(t+1)$$



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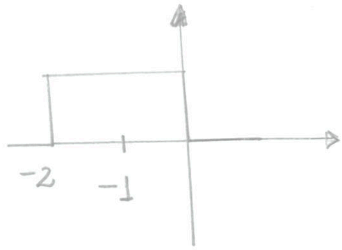
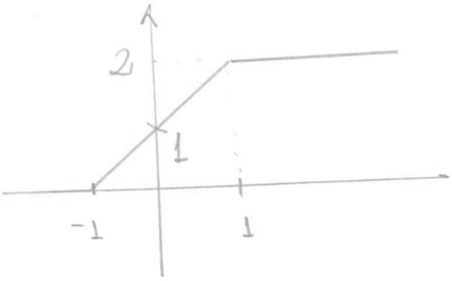
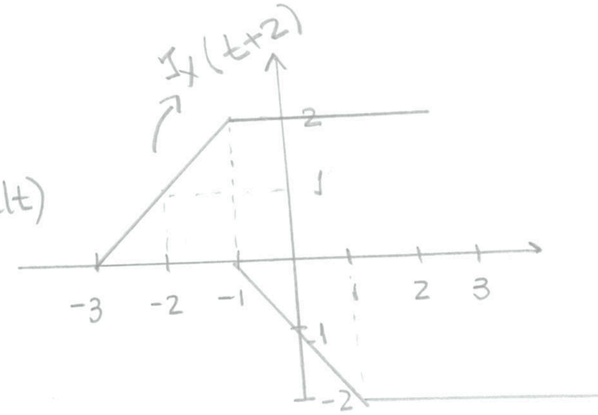


$$= \mu(t+2) - \mu(t)$$

$I_x(t)$

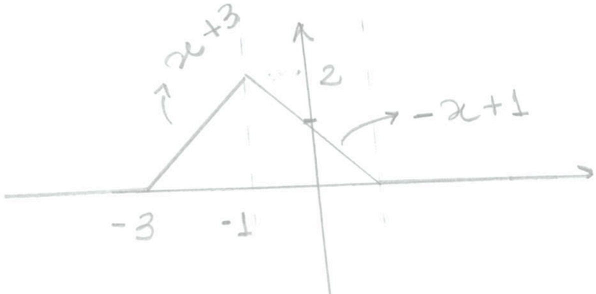


$\xrightarrow{I_x(t+2) - I_x(t)}$



$$\int_{-1}^{t} dt = t+1$$

$=$



$$\boxed{G_2(t) * y(t) = (t+3)G_2(t+2) + (-t+1)G_2(t)}$$

5)

$$y(t) = e^{t} \int_{t-2}^{t} x(\beta)\, e^{-\beta}\, d\beta$$

a) $h(t) = e^{t}\,(u(t) - u(t-2))$

$$h(t) = e^{t}\, G_2(t-1)$$

b) Causal ?

$h(t) < 0 \longrightarrow t < 0$

$\therefore$ causal //

BIBO

é BIBO

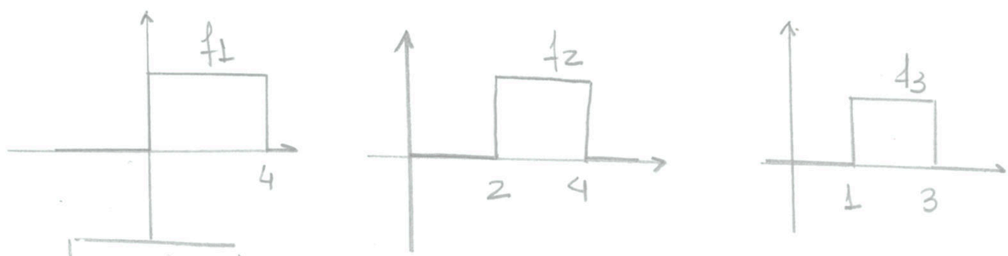
pq p/ $t \to \infty \to h(t) = \emptyset$, pois temos um gate

6)

$f_1 = G_4(t-2)$

$f_2 = G_2(t-3)$

$f_3 = G_2(t-2)$



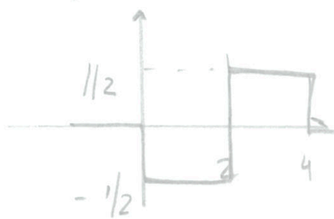
$\boxed{f_1 = g_1}$ //

$$g_2 = f_2 - \frac{\langle g_1 f_2 \rangle}{\langle g_1^2 \rangle}\, g_1 = \boxed{f_2 - \frac{1}{2}\, g_1}$$

$\downarrow$

$\langle g_1 f_2 \rangle = 2$

$\langle g_1^2 \rangle = 4$

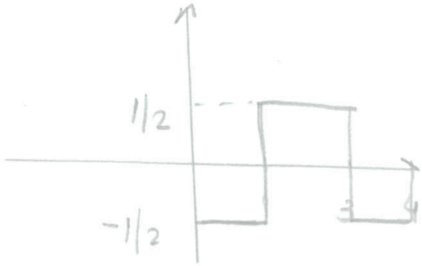


$$g_3 = f_3 - \frac{\langle f_3 g_1 \rangle}{\langle g_1^2 \rangle}\, g_1 - \frac{\langle f_3 g_2 \rangle}{\langle g_2^2 \rangle}\, g_2 \longrightarrow$$

$\langle f_3 g_1 \rangle = 2$

$\langle f_3 g_2 \rangle = \emptyset$

$$\boxed{g_3 = f_3 - \frac{1}{2} g_1}$$



7)

$$\epsilon(t) = \underbrace{(t^2 - 2t)\, G_2(t-1)}_{y(t)} - \Big(a \underbrace{G_2(t-1)}_{x_1} + b \underbrace{t\, G_2(t-1)}_{x_2}\Big)$$

$$\langle x_1^2 \rangle = \int_0^2 dt = 2 \qquad \langle y x_1 \rangle = \int_0^2 (t^2 - 2t)\,dt = \frac{t^3}{3} - t^2 \Big|_0^2 =$$

$$\langle x_1 x_2 \rangle = \int_0^2 t\,dt = 2 \qquad 8/3 - 4 = -4/3$$

$$\langle x_2^2 \rangle = \int_0^2 t^2\,dt = 8/3 \qquad \langle y x_2 \rangle = \int_0^2 t^3 - 2t^2\,dt = \frac{t^4}{4} - \frac{2t^3}{3}\Big|_0^2$$

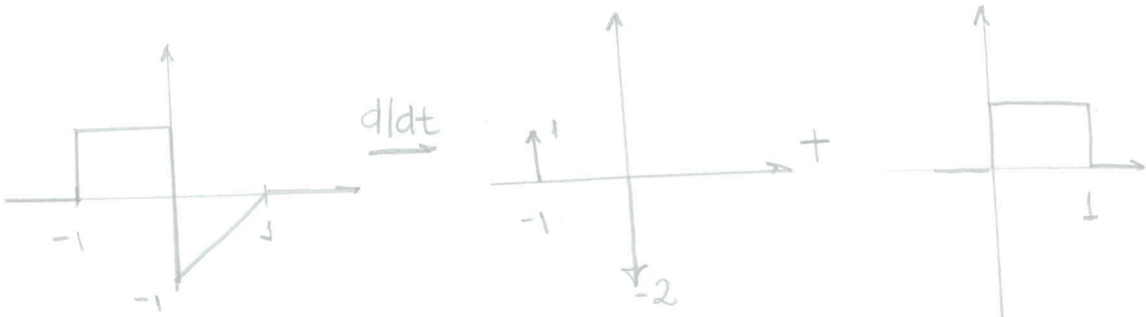
$$4 - \frac{16}{3} = -4/3$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 8/3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -4/3 \\ -4/3 \end{bmatrix} \rightsquigarrow \begin{cases} 2a + 2b = -4/3 \\ 2a + 8/3\, b = -4/3 \end{cases}$$

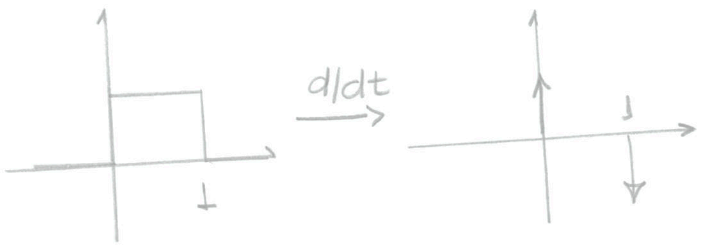
$$\boxed{b = 0} \qquad \boxed{a = -2/3}$$

8)

a) $p(t - k5)$ $\qquad p(t) = G_1(t + 0{,}5) + (t-1)(G_1(t - 0{,}5))$



Derivando



$$\boxed{c_k = \frac{1}{5j\frac{2\pi}{5}k}\left(e^{j\frac{2\pi}{5}k} - 2 + \frac{1-e^{-j\frac{2\pi}{5}k}}{j2\pi/5\,k}\right)}$$

b) $c_0 = \frac{\text{Area}}{5} = \frac{1-1/2}{5} = \boxed{\frac{1}{10}}$

9) $x(t) = 3je^{j\pi t/7} + (2-5j)e^{-j2\pi t/7} + 3\,\text{sen}\,(3\pi t/7)$

a) $T = ?$ $\quad \frac{2\pi}{T_1} = \frac{\pi}{7} \longrightarrow T_1 = 14 \qquad \frac{2\pi}{T_2} = \frac{2\pi}{7} \to T_2 = 7$

$$\frac{2\pi}{T_3} = \frac{3\pi}{7} \to T_3 = \frac{14}{3}$$

$$14\cdot \underset{\substack{\uparrow\\1}}{p} = 7\underset{\substack{\uparrow\\2}}{q} = \frac{14}{3}\underset{\substack{\uparrow\\3}}{\omega} = T \longrightarrow \boxed{T=14}$$

b) $\frac{2\pi}{14} \cdot 1 = \frac{\pi}{7}$ $\quad \frac{2\pi}{14} \cdot 2 = \frac{2\pi}{7}$ $\quad \frac{2\pi}{14} \cdot 3 = \frac{3\pi}{7}$

$$x(t) = 3j e^{j\frac{2\pi}{14}(1)t} + (2-5j)e^{-j\frac{2\pi}{14}(2)t} + \frac{3}{2j}\left(e^{j\frac{2\pi}{14}(3)t} - e^{-j\frac{2\pi}{14}(3)t}\right)$$

$\boxed{C_1 = 3j}$ $\quad \boxed{C_2 = (2-5j)}$

Demais Nulos $\quad \boxed{C_3 = 3/2j}$ $\quad \boxed{C_{-3} = -3/2j}$

c) $P_m = \sum |c_k|^2 = 9 + 29 + 9/4 + 9/4 = 38 + 9/2 = \boxed{85/2}$

10 a) $C_0 = \frac{1}{5}\left[\int_{-1}^{0}(-t^2+1)\,dt + \int_{0}^{1}(t-1)\,dt\right]$

$$\frac{1}{5}\left[-t^3/3 + t\Big|_{-1}^{0} + \frac{t^2}{2} - t\Big|_{0}^{1}\right] = \frac{1}{5}\left[-(1/3 - 1) + \frac{1}{2} - 1\right]$$

$$C_0 = \frac{1}{5}\left[-1/3 + 1/2\right] = \frac{1}{5} \cdot \frac{-2+3}{6} = \frac{1}{30}$$

$\boxed{C_0 = 1/30}$

b) $$P_m = \frac{1}{5}\left[\int_{-1}^{0}\underbrace{(-t^2+1)^2}_{t^4-2t^2+1}dt + \int_0^1 \underbrace{(t-1)^2}_{t^2-2t+1}dt\right.$$

$$\frac{1}{5}\left[\left(t^5/5 - 2t^3/3 + t\right)\Big|_{-1}^{0} + \left(t^3/3 - t^2 + t\right)\Big|_0^1\right]$$

$$\frac{1}{5}\left[-\left(-1/5 + 2/3 - 1\right) + 1/3 - 1 + 1\right] = \frac{1}{5}\left(1/5 - 1/3 + 1\right)$$

$$\frac{1}{5}\left(\frac{3-5+15}{15}\right) = \boxed{\frac{13}{75}}$$