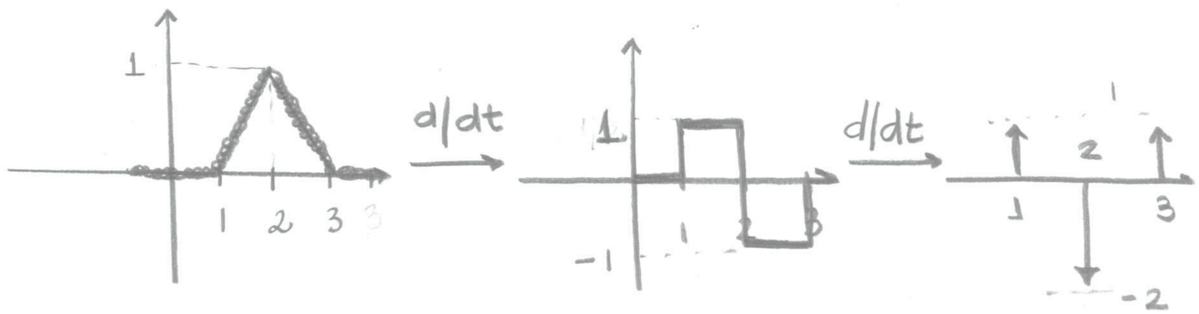
Resolução PR3 - 2015

(1) $X(\omega) = \mathcal{F}\{ e^{j2t} TRI_2(t-2) \}$

$$TRI_{2T}(t) = (t/T + 1)\, G_T(t + T/2) + (-t/T + 1)\, G_T(t - T/2)$$

$$\mathcal{F}\{ x(t) e^{j\omega_0 t} \} = X(\omega - \omega_0)$$



$$X(\omega) = \frac{e^{-j\omega} - 2e^{-2j\omega} + e^{-3j\omega}}{(j\omega)^2}$$

$$\mathcal{F}\{ e^{j2t} TRI_2(t-2) \} = X(\omega - 2) = \boxed{\frac{e^{-j(\omega-2)} - 2e^{-2j(\omega-2)} + e^{-3(\omega-2)j}}{-(\omega-2)^2}} //$$

ou

$$\mathcal{F}\{ TRI_2(t-2) \} = Sa^2(\omega/2)\, e^{-2j\omega}$$

$$\mathcal{F}\{ e^{j2t} TRI(t-2) \} = X(\omega-2) = \boxed{Sa^2\left(\frac{\omega-2}{2}\right) e^{-2j(\omega-2)}} //$$

② $\mathcal{F}\{te^{-t}u(t)\}$

Propriedade

$\rightarrow \mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{j\omega + a}$

$\rightarrow \mathcal{F}\{t^m x(t)\} = j^m \dfrac{d^m}{d\omega^m} X(\omega)$

$$\mathcal{F}\{t\,x(t)\} = j \cdot \frac{d}{d\omega} X(\omega)$$

$$x(t) = e^{-t}u(t) \rightarrow X(\omega) = \frac{1}{j\omega + 1}$$

$$\mathcal{F}\{te^{-t}u(t)\} = j \cdot \frac{d}{d\omega}\left(\frac{1}{j\omega+1}\right)$$

$$j \cdot \frac{(-j)}{(j\omega+1)^2} = \frac{1}{-\omega^2 + 2j\omega + 1} = \frac{-1}{\omega^2 - 2j\omega - 1}$$

$$\boxed{\mathcal{F}\{te^{-t}u(t)\} = \frac{-1}{\omega^2 - 2j\omega - 1}}$$

③ $I = \int_{-\infty}^{\infty} Sa(2t)\,x(t)\,dt \qquad \mathcal{F}\{x(t)\} = \omega^2 G_4(\omega)$

Propriedades

$$\int_{-\infty}^{\infty} x(t)\,dt = X(0)$$

$$\mathcal{F}\{x(t) \cdot y(t)\} = \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$I = \frac{1}{2\pi}\, \mathcal{F}\{Sa(2t)\} * \mathcal{F}\{x(t)\} \qquad \mathcal{F}\{Sa(\tfrac{\omega_0 T}{2})\} = \frac{2\pi}{\omega_0} G_{\omega_0}(\omega)$$

$$I = \frac{1}{2\pi} \cdot \left(\frac{2\pi}{4} \cdot G_4(\omega)\right) * \left(\omega^2 G_4(\omega)\right)\Big|_{\omega=0}$$

$$I = \frac{1}{2\pi} \cdot \frac{2\pi}{4} \left[\int_{-\infty}^{\infty} G_4(\beta) \cdot (\omega-\beta)^2 G_4(\omega-\beta)\, d\beta\right]\Big|_{\omega=0}$$

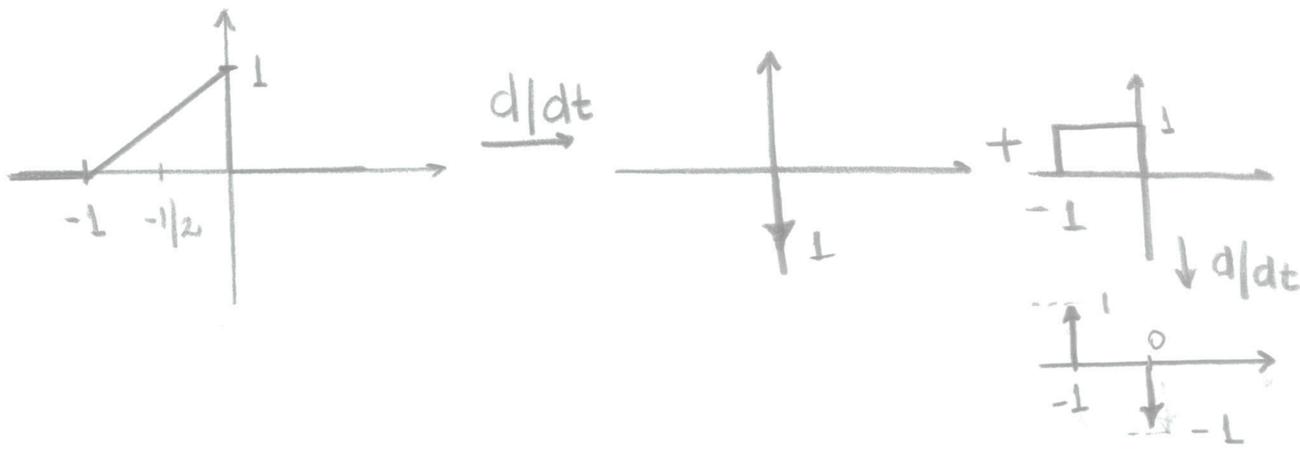
$$I = \frac{1}{4} \int_{-\infty}^{\infty} G_4(\beta)\,(\beta^2)\, G_4(-\beta)\, d\beta$$

$$I = \frac{1}{4} \int_{-2}^{2} \beta^2\, d\beta = \frac{1}{4} \left(\beta^3/3\right)\Big|_{-2}^{2}$$

$$I = \frac{1}{4}\left(8/3 + 8/3\right) = \boxed{\frac{4}{3}}$$

(4) $x(t) = ?$ $\quad X(\omega) = (\omega+1)\, G_1(\omega+0{,}5)$

$$\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$$



$$\mathcal{F}\{X(t)\} = \frac{1}{j\omega}\left(-1 + \frac{e^{j\omega} - 1}{j\omega}\right) = 2\pi\, x(-\omega)$$

$$x(-\omega) = \frac{1}{2\pi j\omega}\left(-1 + \frac{e^{j\omega} - 1}{j\omega}\right)$$

$$\downarrow$$

$$\boxed{X(t) = \frac{1}{2\pi j t}\left(1 + \frac{e^{-jt} - 1}{+jt}\right)}$$

(5) $x(t) = Sa(5t)\, Sa(4t)\, Sa(3t)$

a) $X(\omega) = \frac{1}{2\pi}\left(\pi/5\, G_{10}(\omega)\right) * \left(\pi/4\, G_8(\omega)\right) * \left(\pi/3\, G_6(\omega)\right)$

$$\omega_{máx} = \frac{10+8+6}{2} = 12 \qquad B = \frac{\omega_{máx}}{2\pi}$$

$$B = \frac{12}{2\pi} = \frac{6}{\pi}$$

$$T_{max} < \frac{1}{2B} \rightarrow \boxed{T < \frac{\pi}{12}}$$

b) $\omega_{max} = \pi/5$ rad/s

$$x_a(t) = \sum x(k3)\, p(t-k3) \qquad p(t) = TRI_1(t)$$

$$H(j\omega) = \frac{T\, G_{\omega_0}(\omega)}{P(\omega)} \qquad T = 3 \rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

$P(\omega) = \mathcal{L}\{TRI_1(t)\}$ pelo propriedade

$$\mathcal{L}\{TRI_{2T}(t)\} = T\,Sa^2\left(\frac{\omega T}{2}\right)$$

$$P(\omega) = \frac{1}{2}\,Sa^2\left(\frac{\omega}{4}\right)$$

$$H(j\omega) = \frac{3 \cdot G_{2\pi/3}(\omega)}{1/2\, Sa^2(\omega/4)} \rightarrow \boxed{H(j\omega) = \frac{6 \cdot G_{2\pi/3}(\omega)}{Sa^2(\omega/4)}}$$

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$$x(t) = t^2 e^{2t} u(-t)$$

$$\mathcal{L}\{t^m/m!\, e^{-at} u(t)\} = \frac{1}{(s+a)^{m+1}}$$

$$Re(s+a) > 0$$

$$x(-t) = y(t) = t^2 e^{-2t} u(t)$$

$$\mathcal{L}\{x(-t)\} = X(-s)$$

$$Y(s) = \frac{1}{(s+2)^3} \cdot 2! = \frac{2}{(s+2)^3}$$

$$Y(s) = X(-s) \rightarrow X(s) = \frac{2}{(-s+2)^3} = \frac{-2}{(s-2)^3}$$

$$Re(s) < 2$$

$$\boxed{X(s) = \frac{-2}{(s-2)^3} \quad Re(s) < 2}$$

7 $X(s) = \dfrac{5s-17}{(s-1)(s-5)}$ $\quad 1 < Re(s) < 5$

$$X(s) = \frac{A}{s-1} + \frac{B}{s-5} = \underbrace{\frac{3}{s-1}}_{Re>1} + \underbrace{\frac{2}{s-5}}_{Re<5}$$

$$\mathcal{L}\left\{\frac{t^m}{m!} e^{-at} u(t)\right\} = \frac{1}{(s+a)^{m+1}} \quad Re(s+a) > 0$$

$$\mathcal{L}\left\{\frac{(-t)^m}{m!} e^{at} u(-t)\right\} = \frac{1}{(-s+a)^{m+1}} \quad Re(s+a) < 0$$

$$\boxed{x(t) = 3 . e^{+t} u(t) - 2 e^{5t} u(-t)}$$

8 $I = \displaystyle\int_{-\infty}^{\infty} t\, h(t)\, dt$ $\quad h(t) = e^{-2t} \cos(3t)\, u(t)$

$$I = X(s)\Big|_{s=0} \rightarrow X(s) = \mathcal{L}\{t\, h(t)\} = -\frac{d}{ds} H(s)\Big|_{s=0}$$

Propriedade

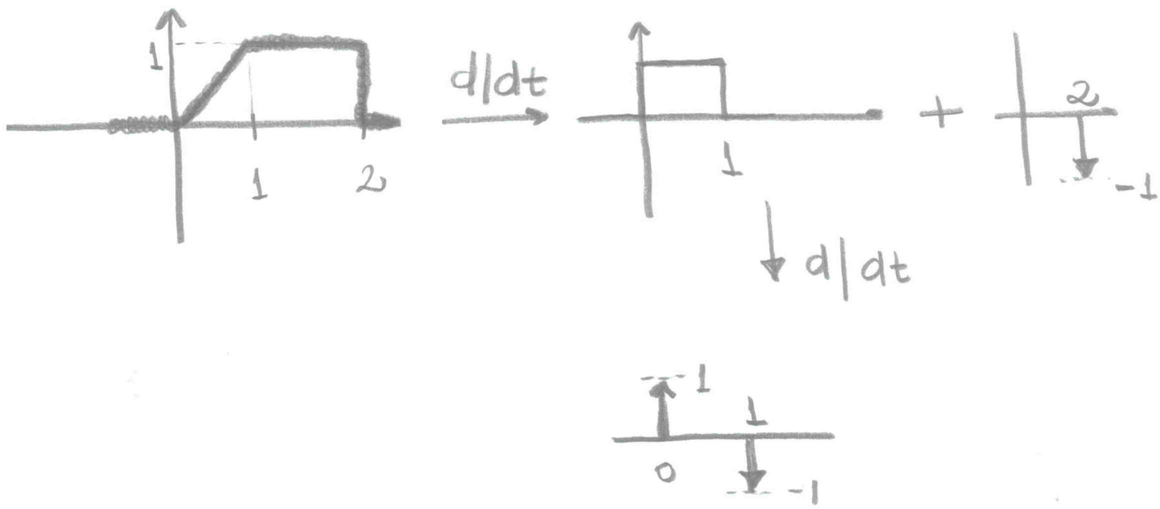
$$\mathcal{L}\{e^{-at} \cos(\beta t)\, u(t)\} = \frac{s+a}{(s+a)^2+\beta^2}$$

$$H(s) = \frac{s+2}{(s+2)^2+9} = \frac{s+2}{s^2+4s+13}$$

$$-\frac{d}{ds}H(s)\Big|_{s=0} = -\frac{[(s^2+4s+13)-(s+2)(2s+4)]}{(s^2+4s+13)^2}\Big|_{s=0}$$

$$-\frac{d}{ds}H(s)\Big|_{s=0} = -\frac{((13)-(2)(4))}{13^2} = \boxed{-\frac{5}{13^2}}$$

(9) $x(t) = t\,G_1(t-0{,}5) + G_1(t-1{,}5)$



$$\boxed{X(s) = \frac{1}{s}\left(-e^{-2s} + \frac{1-e^{-s}}{s}\right) \qquad Re(s) \neq 0}$$

(10) $(p^3\,4L_1C_2L_3 + p^2\,16L_1C_2R + p(2L_1+L_3) + 4R)y = 4Rx$

$H(s) = \frac{1}{D(\lambda)}$ $\quad D(\lambda) = \lambda^3 + 2\lambda^2 + 2\lambda + 1$ $\quad \lambda = \frac{s}{\omega_c}$

$$\frac{y}{x} = \frac{4R}{p^3 4L_1C_2L_3 + p^2\, 16L_1C_2R + p\,(2L_1+L_3) + 4R}$$

Dividindo em cima e embaixo por $4L_1C_2L_3$

$$\frac{Y}{X} = \frac{4R/4L_1C_2L_3}{p^3 + p^2\frac{4R}{L_3} + p\left(\frac{2L_1+L_3}{4L_1C_2L_3}\right) + \frac{R}{L_1C_2L_3}}$$

$$\frac{4R/4L_1C_2L_3}{\lambda^3 + \lambda^2\frac{4R}{L_3} + \lambda\left(\frac{2L_1+L_3}{4L_1C_2L_3}\right) + \frac{R}{L_1C_2L_3}} = \frac{\omega_c^3}{s^3 + 2s^2\omega_c + 2s\omega_c^2 + \omega_c^3}$$

$$\begin{cases} \omega_c^3 = \dfrac{R}{L_1C_2L_3} \longrightarrow L_1C_2L_3 = \dfrac{R}{\omega_c^3} \longrightarrow C_2 = \dfrac{\cancel{R}}{\omega_c^{\cancel{3}}}\cdot\dfrac{\cancel{\omega_c}}{3\cancel{R}}\cdot\dfrac{\cancel{\omega_c}}{2R} \longrightarrow \boxed{C_2 = \dfrac{1}{6R\omega_c}} \\ 2\omega_c = \dfrac{4R}{L_3} \longrightarrow \boxed{L_3 = \dfrac{2R}{\omega_c}} \\ 2\omega_c^2 = \dfrac{2L_1+L_3}{4L_1C_2L_3} \end{cases}$$

$$2\omega_c^2 \cdot 4 \cdot \frac{R}{\omega_c^3} = 2L_1 + \frac{2R}{\omega_c}$$

$$\frac{8R}{\omega_c} - \frac{2R}{\omega_c} = 2L_1$$

$$\boxed{L_1 = \frac{3R}{\omega_c}}$$