Resolução Comentada PR1 2º Semestre 2017

Questão 1:

$$y_f(t) = ? \longrightarrow x(t) = 5 + 5\cos(300t)$$

$$\downarrow$$

$$x(t) = \underbrace{5e^{0t}}_{DC} + 5\cos(\underbrace{300t}_{\omega})$$

Analisando os gráficos de magnitude e fase conduz-se que:

Na parte constante (DC) $\nearrow M_{dB} = 20dB \rightarrow 10$ em decimal
$\searrow \phi = 0°$

Em $\omega = 300$ $\nearrow M_{dB} = -20dB \longrightarrow 0{,}1$ decimal
$\searrow \phi \cong -157{,}5°$

$$y_f(t) = 5.10 + 5.\ 0.1.\ \cos(300t - 157{,}5°)$$

$$\boxed{50 + 0.5\cos(300t - 157{,}5°)}$$

Questão 2:

$y_u(t) = ?$ (condições iniciais nulas) $\begin{cases} \ddot{y}(0) = \dot{y}(0) = y(0) = 0 \\ \ddot{x}(0) = \dot{x}(0) = x(0) = 0 \end{cases}$

$$\ddot{y} + 4\dot{y} + 13y = 7\ddot{x} + 15\dot{x} + 65x$$

Aplicando Laplace $\begin{cases} \mathcal{L}\{\ddot{y}\} = s^2 Y(s) \\ \mathcal{L}\{\dot{y}\} = sY(s) \end{cases}$ condições iniciais nulas

$$s^2 Y(s) + 4sY(s) + 13Y(s) = 7s^2 X(s) + 15sX(s) + 65X(s)$$

$$(s^2 + 4s + 13)Y(s) = (7s^2 + 15s + 65)X(s)$$

$$Y(s) = \frac{7s^2 + 15s + 65}{s^2 + 4s + 13} X(s)$$

Resposta ao degrau $X(s) = 1/s$

$$Y_u(s) = \frac{1}{s} \frac{7s^2 + 15s + 65}{s^2 + 4s + 13} \overset{!}{=} \frac{1}{s} \cdot \frac{7s^2 + 15s + 65}{(s+2)^2 + 3^2}$$

Frações Parciais

$$Y_u(s) = \frac{\overset{=5}{A}}{s} + \frac{Bs + C}{(s+2)^2 + 3^2} \overset{!}{=} \frac{5(s^2 + 4s + 13) + Bs^2 + Cs}{s(s^2 + 4s + 13)}$$

s^2: $5 + B = 7$

$B = 2$

s: $20 + C = 15$

$C = -5$

$$Y_u(s) = \frac{5}{s} + \frac{2s - 5}{(s+2)^2 + 3^2}$$

Transformadas:

$$\mathcal{L}\{\mu(t)\} = 1/s$$

$$\mathcal{L}\{\cos(\beta t)\underset{}{\mu(t)}\} = \frac{s}{s^2+\beta^2}$$

$$\mathcal{L}\{\sin(\beta t)\mu(t)\} = \frac{\beta}{s^2+\beta^2}$$

$$\mathcal{L}\{e^{-at}x(t)\} = X(s+a)$$

$$Y\mu(s) = \frac{5}{s} + \frac{2s-5}{(s+2)^2+3^2}$$

$$= \underbrace{\frac{5}{s}}_{5\mu(t)} + \underbrace{\frac{2(s+2)}{(s+2)^2+3^2}}_{2e^{-2t}\cos(3t)} \underbrace{- \frac{9}{(s+2)^2+3^2}}_{-3e^{-2t}\sin(3t)}$$

$$\boxed{y\mu(t) = \left(5 + 2e^{-2t}\cos(3t) - 3e^{-2t}\sin(3t)\right)\mu(t)}$$

Questão 3:

a) $Y(s) = \mathcal{L}\{y(t)\} = ?$

$$\ddot{y} + 6\dot{y} + 8y = 0 \qquad y(0), \dot{y}(0) \text{ dados}$$

Transformadas

$\mathcal{L}\{\ddot{y}\} = s^2Y(s) - s\,y(0) - \dot{y}(0)$

$\mathcal{L}\{\dot{y}\} = sY(s) - y(0)$

$$s^2Y(s) - s\,y(0) - \dot{y}(0) + 6(sY(s) - y(0)) + 8Y(s) = 0$$

$$(s^2 + 6s + 8)Y(s) = y(0)(s+6) + \dot{y}(0)$$

$$\boxed{Y(s) = \frac{y(0)(s+6) + \dot{y}(0)}{s^2 + 6s + 8}}$$

b) $\dot{y}(0)\,(y(0)) \longrightarrow y(t) = y(0)e^{-4t}u(t)$

$$\downarrow$$

$$Y(s) = \frac{y(0)}{(s+4)} \quad (1)$$

$$Y(s) = \frac{y(0)(s+6) + \dot{y}(0)}{(s+4)(s+2)} \xrightarrow[\text{parciais}]{\text{frações}} Y(s) = \frac{A}{s+4} + \frac{B}{s+2} \quad (2)$$

Comparando (1) e (2)

$$\begin{cases} A = \dfrac{2y(0) + \dot{y}(0)}{-2} \\ B = \dfrac{4y(0) + \dot{y}(0)}{2} \end{cases}$$

$A = -2y(0) = 2y(0) + \dot{y}(0)$

$\hookrightarrow \boxed{\dot{y}(0) = -4y(0)}$

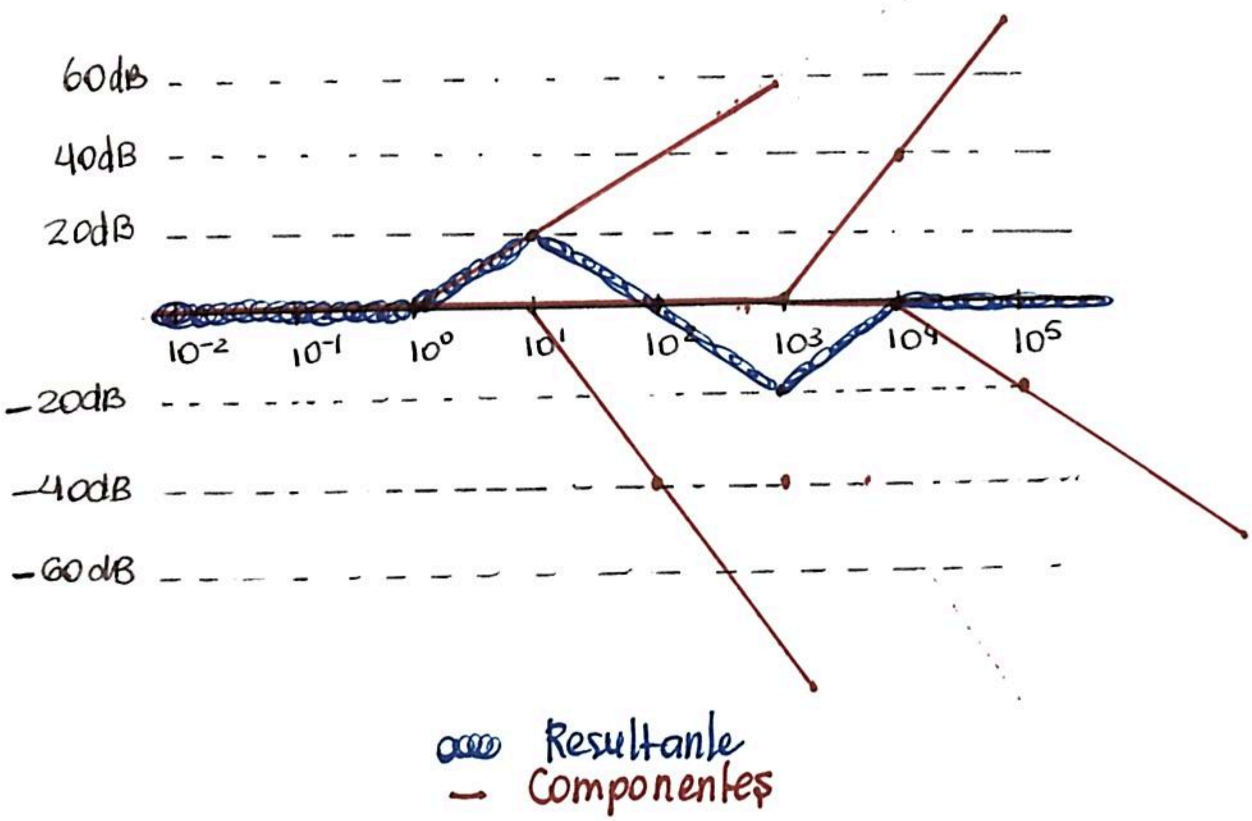
$B = 0$

Questão 4:

a) Ganho DC = 0 dB assíntotas do módulo

$$H(s) = \frac{(s+1)(s+10^3)^2}{(s+10)^2(s+10^4)}$$

$$= \frac{s+1}{1} \cdot \frac{(s+10^3)^2}{10^6} \cdot \frac{10^2}{(s+10)^2} \cdot \frac{10^4}{(s+10^4)}$$



b) $y_f(t) = ?$ $x(t) = \cos(\underbrace{10}_{\omega}t) + \sin(\underbrace{1000}_{\omega}t)$

$\omega = 10 \rightarrow M_{dB} = 20\,dB \rightarrow 10$ decimal
$\rightarrow \phi = 0°$

$\omega = 1000 \rightarrow M_{dB} = -20\,dB \rightarrow 0{,}1$ decimal
$\rightarrow \phi = 0°$

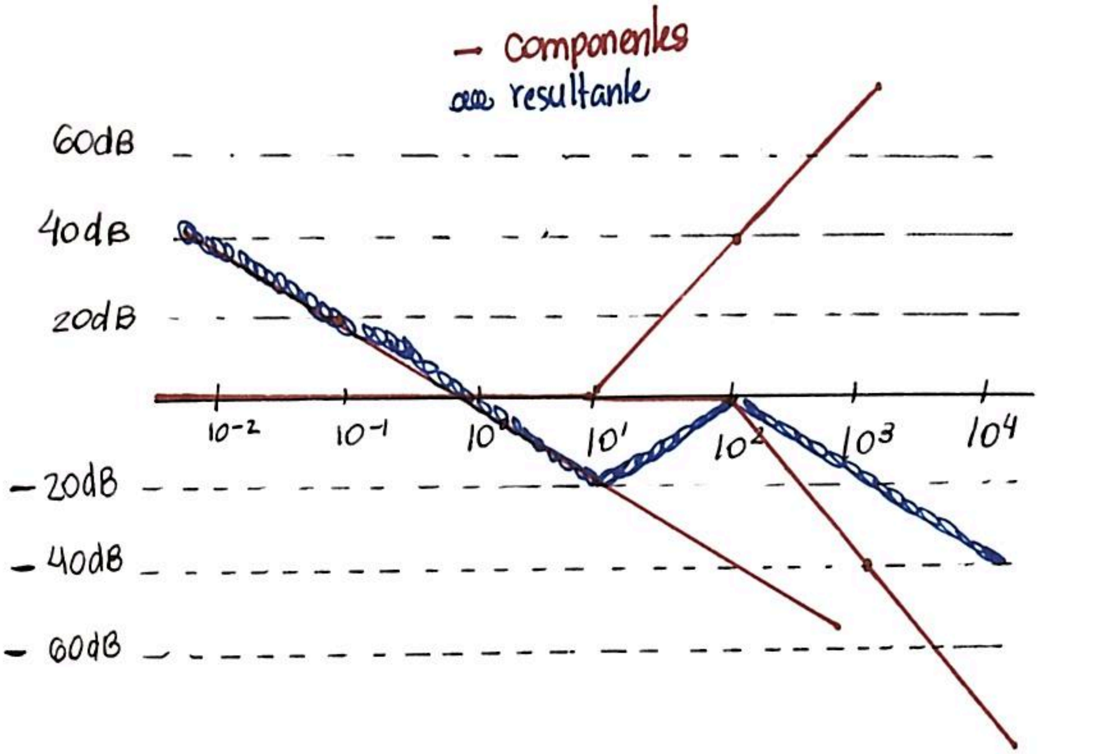
$$\boxed{y_f(t) = 10\cos(10t) + 0{,}1\sin(1000t)}$$

Questão 5:

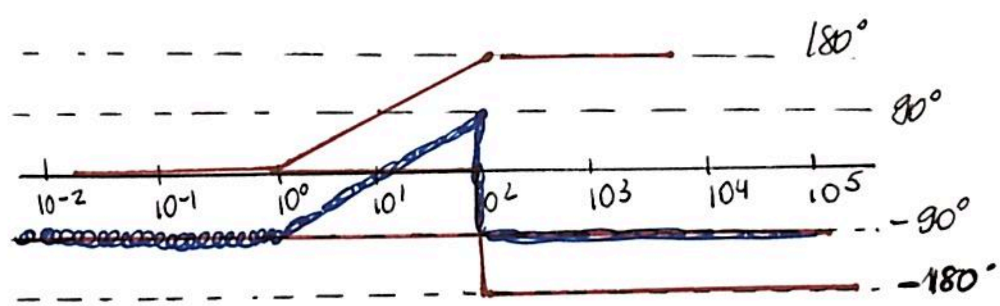
a) Assíntotas do Módulo

$$H(s) = \frac{100(s+10)^2}{s(s^2+20s+100^2)}$$

$$= \underbrace{\frac{(s+10)^2}{10^2}}_{0 \to 180^\circ} \cdot \underbrace{\frac{10}{s}}_{\phi \to -90^\circ} \cdot \underbrace{\frac{100^2}{(s^2+20s+100^2)}}_{\omega_n = 100} \nearrow -180^\circ$$



b)



Questão 6:

$y_f(t) = ? \longrightarrow x(t) = t$

$$H(s) = \frac{1}{s^2(s+1)}$$

Podemos resolver usando o método coeficientes a determinar

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2(s+1)}$$

$$\downarrow$$

$$p^2(p+1)\, y = x \rightarrow \underbrace{p^2(p+1)}_{D(p)} y = t$$

$D(p) = p^2(p+1)$ → $\alpha = 0$ mult. 2, $\alpha = -1$

$\bar{D}(p) = p^2$ → $\gamma = 0$ mult. 2

ATENÇÃO que temos modos iguais

$$y_f(t) = Ct^2 + Dt^3$$

$$(p^3 + p^2)(Ct^2 + Dt^3) = t$$

$$2C + 6Dt + 6D = t$$

$$\begin{cases} 6D = 1 \rightarrow D = 1/6 \\ 2C + 6D = 0 \rightarrow C = -\dfrac{1}{2} \end{cases}$$

$$\boxed{y_f(t) = -\frac{1}{2}t^2 + \frac{1}{6}t^3}$$

Questão f:

Equação diferencial homogênea e as condições iniciais

$$y(t) = 2e^{-2t} + e^{-t}(\cos(2t) - 5\sin(2t))$$

Modos próprios

$$2e^{-2t} \longrightarrow p+2$$

$$e^{-t}(\cos(2t) - 5\sin(2t)) \rightarrow (p+1)^2 + 2^2$$

$$\boxed{(p+2)((p+1)^2+2^2)\,y = 0}$$

$\hookrightarrow y(t) = Ae^{-2t} + Be^{-t}\cos(2t) + Ce^{-t}\sin(2t)$

3 condições iniciais

$$\left\{\begin{array}{l} \boxed{y(0) = 3} \\ \dot{y}(t) = -4e^{-2t} - e^{-t}(\cos(2t) - 5\sin(2t)) \\ \qquad -2e^{-t}\sin(2t) - 10e^{-t}\cos(2t) \end{array}\right.$$

$\hookrightarrow \boxed{\dot{y}(0) = -15}$

$$\ddot{y}(t) = 8e^{-2t} + e^{-t}(\cos(2t) - 5\sin(2t))$$
$$+2e^{-t}\sin(2t) + 10e^{-t}\cos(2t)$$
$$+2e^{-t}\sin(2t) - 4e^{-t}\cos(2t)$$
$$+10e^{-t}\cos(2t) + 20e^{-t}\sin(2t)$$

$\hookrightarrow \boxed{\ddot{y}(0) = 25}$

Questão 8:

$y(t) = ?$ $(p^2+4)\,y = 8\cos(2t) - 12\sin(2t)$

$y(0) = 10,\ \dot{y}(0) = 11$

$D(p) = p^2+4 \rightarrow \alpha = \pm 2j$

$\bar{D}(p) = p^2+4 \rightarrow \gamma = \pm 2j$

MODOS IGUAIS

$y_f(t) = t\,(A\sin(2t) + B\cos(2t))$

$y_h(t) = C\sin(2t) + D\cos(2t)$

$(p^2+4)\,(t\,(A\sin(2t) + B\cos(2t))) = 8\cos(2t) - 12\sin(2t)$

$\cancel{4t\,(A\sin(2t) + B\cos(2t))} +$

$+ 4A\cos(2t) - \cancel{4tA\sin(2t)} -$

$- 4B\sin(2t) - \cancel{4Bt\cos(2t)}$

$4A = 8 \rightarrow A = 2$

$-4B = -12 \rightarrow B = 3$

obs

$p(tA\sin(2t) + Bt\cos(2t)) =$

$$= \left.\begin{aligned} &A\sin(2t) + \\ &+ 2tA\cos(2t) + \\ &B\cos(2t) + \\ &- 2Bt\sin(2t) \end{aligned}\right\} (1)$$

$p(1) = 2A\cos(2t)$

$+ 2A\cos(2t) - 4tA\sin(2t)$

$- 2B\sin(2t) -$

$- 2B\sin(2t) -$

$- 4Bt\cos(2t)$

$y(t) = t\,(2\sin(2t) + 3\cos(2t)) +$

$+ C\sin(2t) + D\cos(2t)$

$y(0) = 10 = D$

$\dot{y}(t) = (2\sin(2t) + 3\cos(2t)) + t\,(4\cos(2t) - 6\sin(2t))$

$+ 2C\cos(2t) - 2D\sin(2t) \rightarrow \dot{y}(0) = 11 \rightarrow C = 4$

$$\boxed{y(t) = 10\cos(2t) + 4\sin(2t) + t\,(3\cos(2t) + 2\sin(2t))}$$

Questão 9 (Matéria P2)

Questão 10 (Matéria P2)

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Questão 9:

a) $Y(z) = ?$

$$y[n+2] + 6y[n+1] + 8y[n] = -2(-3)^n \quad \begin{cases} y[0] = 6 \\ y[1] = -20 \end{cases}$$

Transformadas Z necessárias:

$$\mathcal{Z}\{y[n+2]\} = z^2Y(z) - z^2y[0] - zy[1]$$
$$\mathcal{Z}\{y[n+1]\} = zY(z) - zy[0]$$
$$\mathcal{Z}\{y[n]\} = Y(z)$$
$$\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$$

Aplicando as transformadas temos:

$$z^2Y(z) - z^2\underbrace{y[0]}_{6} - z\underbrace{y[1]}_{-20} + 6(zY(z) - z\underbrace{y[0]}_{6}) + 8Y(z) = \frac{-2}{z+3}$$

$$Y(z)(z^2+6z+8) = \frac{-2z}{z+3} + 6z^2 - 20z + 36z$$

$$= \frac{-2z + (z+3)(6z^2+16z)}{(z+3)}$$

$$Y(z) = \frac{-2z + 6z^3 + 16z^2 + 18z^2 + 48z}{(z+3)(z^2+6z+8)}$$

$$\boxed{Y(z) = \frac{6z^3 + 34z^2 + 46z}{(z+3)(z+2)(z+4)}}$$

b) $Y(z) \rightarrow y[n] = ?$

$$Y(z) = \frac{6z^3 + 34z^2 + 46z}{(z+4)(z+3)(z+2)}$$

$\downarrow$

<u>Frações Parciais</u>

$$\frac{Y(z)}{z} = \frac{6z^2 + 34z + 46}{(z+4)(z+3)(z+2)} = \frac{A}{(z+4)} + \frac{B}{(z+3)} + \frac{C}{(z+2)}$$

$$A = \frac{6.(-4)^2 + 34(-4) + 46}{(-4+3)(-4+2)} = 3$$

$$B = \frac{6(-3)^2 + 34(-3) + 46}{(-3+4)(-3+2)} = 2$$

$$C = \frac{6(-2)^2 + 34(-2) + 46}{(-2+4)(-2+3)} = 1$$

<u>Utilizando</u>

$$\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$$

$$Y(z) = \frac{3z}{(z+4)} + \frac{2z}{(z+3)} + \frac{z}{(z+2)}$$

$$\boxed{y[n] = \left(3 \cdot (-4)^n + 2(-3)^n + (-2)^n\right) u[n]}$$

Questão 10:

a) $y_f[n] = ?$

$$\underbrace{(p^2+4p+3)}_{(p+1)(p+3)} y[n] = -8(-1)^n \quad [1]$$

$\alpha = -1$ (mult. 1) $\quad \alpha = -3$

$\gamma = -1$ (mult. 1)

porém resposta homogênea e forçada possuem mesma raiz

$$y_f[n] = An(-1)^n \quad [2]$$

substituindo [2] → [1]

$$(p^2+4p+3)A_n(-1)^n = -8(-1)^n$$

$$A(n+2)(-1)^{n+2} + 4A(n+1)(-1)^{n+1} + 3nA(-1)^n = -8(-1)^n$$

$$A(n+2)(-1)^n - 4A(n+1)(-1)^n + 3An(-1)^n = -8(-1)^n$$

$$(An - 4An + 3An)(-1)^n + (2A-4A)(-1)^n = -8(-1)^n$$

$$2A - 4A = -8$$

$$\boxed{A = +4}$$

$$\boxed{y_f[n] = 4n(-1)^n}$$

b) Solução $y[n] = ?$

$$y[n] = y_h[n] + y_f[n]$$

$$y_h[n] = B(-1)^n + C(-3)^n$$

$$y[n] = B(-1)^n + C(-3)^n + 4n(-1)^n \quad [3]$$

Substituindo as condições iniciais $y[0] = 5$ e $y[1] = -15$ em [3]:

$$y[0] = 5 = B + C$$

$$y[1] = -15 = -B - 3C - 4$$

$+$

$$-2C = -6$$

$$\boxed{C = 3} \longrightarrow \boxed{B = 2}$$

$$\left[y[n] = 2(-1)^n + 3(-3)^n + 4n(-1)^n \right]$$