Resolução P2 - 1S 2018:

Questão 1:

a) $\dot{v} = (v-1)(v+2) = v^2 + v - 2$

pontos de equilíbrio?

$$\dot{v} = 0 \rightarrow (v-1)(v+2) = 0$$

$$\boxed{v = 1} \qquad \boxed{v = -2}$$

pontos de equilíbrio

b) Aprox. linear

$$\dot{v} = \frac{\partial \dot{v}}{\partial v} \cdot v \longrightarrow \dot{v} = [2v+1] \cdot v$$

Em $v = 1$

$$\dot{v} = \underbrace{[3]}_{A} v$$

autovalor $\det(sI - A) = 0$

$s - 3 = 0$

$s = 3 \rightarrow$ parte real positiva (INSTÁVEL)

Em $v = -2$

$$\dot{v} = [-3] v$$

$\det(s+3) = 0$

$s = -3$

Assintoticamente Estável.

Questão 2:

$$\dot{v}_1 = -v_1(v_2 - 1) + 3x^2$$
$$\dot{v}_2 = (v_1 + 1)v_2 - 2x$$

a) Pontos de equilíbrio? p/ $x = 0$

$$\dot{v}_1 = 0 \longrightarrow -v_1(v_2 - 1) = 0 \quad \begin{cases} v_1 = 0 \\ v_2 = 1 \end{cases}$$
$$\dot{v}_2 = 0 \longrightarrow (v_1 + 1)v_2 = 0 \quad \begin{cases} v_1 = -1 \\ v_2 = 0 \end{cases}$$

Se $v_1 = 0 \rightarrow v_2 = 0 \quad (0,0)$

Se $v_2 = 1 \rightarrow v_1 = -1 \quad (-1,1)$

$$\boxed{\text{Pontos de equilíbrio } (0,0) \text{ e } (-1,1)}$$

b) Jacobiano?

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial \dot{v}_1}{\partial v_1} & \frac{\partial \dot{v}_1}{\partial v_2} \\ \frac{\partial \dot{v}_2}{\partial v_1} & \frac{\partial \dot{v}_2}{\partial v_2} \end{bmatrix}}_{A} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\partial \dot{v}_1}{\partial x} \\ \frac{\partial \dot{v}_2}{\partial x} \end{bmatrix}}_{B} x$$

$$A = \begin{bmatrix} -v_2 + 1 & -v_1 \\ v_2 & v_1 + 1 \end{bmatrix} \qquad B = \begin{bmatrix} 6x \\ -2 \end{bmatrix}$$

para $(0,0)$

$$A\Big|_{0,0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

para $(\underbrace{-1}_{v_1}, \underbrace{1}_{v_2})$

$$A\Big|_{(-1,1)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

os pontos de eq. são para $x=0$, logo

$$b = \begin{bmatrix} 0 \\ -2 \end{bmatrix}.$$

Questão 3: Realização (A, b, c, d)?

$$(p^3 - 4p^2 - 3p - 2)\, y(t) = (5p^3 - 12p^2 - 6p)\, x(t)$$

$$\frac{y(t)}{x(t)} = \frac{5p^3 - 12p^2 - 6p}{p^3 - 4p^2 - 3p - 2}$$

CASO PRÓPRIO

$$\frac{y(t)}{x(t)} = \underbrace{5}_{d} + \frac{+\overbrace{8}^{\bar{\beta}_2}p^2 + \overbrace{9}^{\bar{\beta}_1}p + \overbrace{10}^{\bar{\beta}_0}}{p^3 \underbrace{-4}_{\alpha_2}p^2 \underbrace{-3}_{\alpha_1}p \underbrace{-2}_{\alpha_0}}$$

$$\begin{cases} \bar{\beta}_0 = 10 \\ \bar{\beta}_1 = 9 \\ \bar{\beta}_2 = 8 \end{cases} \qquad \begin{cases} \alpha_0 = -2 \\ \alpha_1 = -3 \\ \alpha_2 = -4 \end{cases}$$

Formas

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} \bar{\beta}_0 & \bar{\beta}_1 & \bar{\beta}_2 \end{bmatrix}$$

$$d = [d]$$

ou

$$A = \begin{bmatrix} 0 & 0 & -\alpha_0 \\ 1 & 0 & -\alpha_1 \\ 0 & 1 & -\alpha_2 \end{bmatrix} \quad B = \begin{bmatrix} \bar{\beta}_0 \\ \bar{\beta}_1 \\ \bar{\beta}_2 \end{bmatrix} \quad C = [0 \;\; 0 \;\; 1]$$

$$d = [d]$$

Ambas são formas equivalentes

$$\boxed{A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ 9 \\ 8 \end{bmatrix} \quad C = [0 \;\; 0 \;\; 1] \quad d = [5]}$$

Questão 4 :

$$\dot{v} = \underbrace{\begin{bmatrix} -7 & 5 \\ -4 & 2 \end{bmatrix}}_{A} v \qquad v(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y = \underbrace{[1 \quad 1]}_{c} v$$

a) $Y(s) = ?$ $\longrightarrow$ $Y(s) = c(sI-A)^{-1}(v(0) + \cancel{b}X(s)^{\nearrow 0}) + D\cancel{X(s)}^{\nearrow 0}$

$$\stackrel{!}{=} c(sI-A)^{-1} v(0)$$

$$\stackrel{!}{=} [1 \quad 1] \begin{bmatrix} s+7 & -5 \\ 4 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Lembrando

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow A^{-1} = \frac{1}{det(A)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$Y(s) = [1 \quad 1] \frac{1}{s^2+5s+6} \underbrace{\begin{bmatrix} s-2 & 5 \\ -4 & s+7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\begin{bmatrix} s+3 \\ s+3 \end{bmatrix}}$$

$$Y(s) = \frac{2s+6}{s^2+5s+6}$$

$$\stackrel{!}{=} \frac{2\cancel{(s+3)}}{(s+2)\cancel{(s+3)}} \rightarrow \boxed{Y(s) = \frac{2}{(s+2)}}$$

b) $Y(s) = \frac{2}{s+2} \longrightarrow y(t) = ?$

<u>Transformada</u>

$y(t) = \mathcal{L}^{-1}\{Y(s)\}$

$\mathcal{L}\{e^{at}u(t)\} = \frac{1}{s-a}$

$$\boxed{y(t) = 2e^{-2t}u(t)}$$

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Questão 5: a e b ?

$$A^{-1} = aI + bA \qquad A = \begin{bmatrix} -7 & 5 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-14+20} \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} 1/3 & -5/6 \\ 2/3 & -7/6 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 & -5/6 \\ 2/3 & -7/6 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} -7b & 5b \\ -4b & 2b \end{bmatrix}$$

$$5b = -5/6 \longrightarrow \boxed{b = -1/6}$$

$$a - 7b = 1/3$$

$$a = 1/3 + 7/6 = \frac{2-7}{6}$$

$$\boxed{a = -5/6}$$

Questão 6:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & -1 \\ -1 & 0 & 3 \end{bmatrix} \qquad \Delta(\lambda) = (\lambda - 2)^3$$

$$M = (A - \lambda I)$$

$$\overset{!}{=} (A - 2I) = \begin{vmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{vmatrix} \rightarrow \underline{\text{rank} = 1}$$

$$\nu = \text{nº colunas} - \text{rank} = 2$$

↑ número de formas de Jordan

$$M^2 = \emptyset \rightarrow k = 2 \leftarrow$$ maior bloco possui dimensão 2

$$\boxed{\text{diag}\,(J_2(2), J_1(2)) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$$

Questão 7: $v(t) = ?$

$$\dot{v} = Av = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} v \qquad v(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda+2 & 1 \\ -1 & \lambda \end{vmatrix}$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda+1)(\lambda+1) = 0$$

$$\begin{cases} e^{\lambda t} = \alpha_0 + \lambda \alpha_1 \\ te^{\lambda t} = \alpha_1 \end{cases} \longrightarrow \begin{matrix} e^{-t} = \alpha_0 - \alpha_1 \\ te^{-t} = \alpha_1 \end{matrix}$$

$$\alpha_0 = e^{-t} + te^{-t}$$

$$e^{At} = \alpha_0 I + \alpha_1 A$$

$$\overset{!}{=} (e^{-t} + te^{-t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{-t} \begin{pmatrix} -2 & -1 \\ 1 & 0 \end{pmatrix}$$

$$e^{At} \overset{!}{=} \begin{bmatrix} e^{-t} - te^{-2t} & -te^{-t} \\ te^{-t} & e^{-t} + te^{-t} \end{bmatrix}$$

$$v(t) = e^{At} v(0)$$

$$\overset{!}{=} \begin{pmatrix} e^{-t} - te^{-2t} & -te^{-t} \\ te^{-t} & e^{-t} + te^{-t} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{v(t) = \begin{bmatrix} e^{-t} - te^{-t} \\ te^{-t} \end{bmatrix}}$$

Questão 8:

a) Forma de Jordan $\hat{A} = ?$

$$A = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-4 & -4 \\ 1 & \lambda \end{vmatrix} = \lambda^2 - 4\lambda + 4$$

$$\downarrow = (\lambda - 2)^2$$

$$M = (A - \lambda I) = (A - 2I) = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$$

$\hookrightarrow$ rank $= 1$

null space = nº colunas - rank

$\downarrow = 2 - 1 = 1$

1 bloco de jordan

$$\text{diag}(J_2(2)) = \hat{A} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\boxed{\hat{A} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}}$$

$\rightarrow \quad (A - \lambda I)\, v_1 = 0$

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = 0$$

$$2v_{11} + 4v_{21} = 0$$

$$v_{11} = 1 \rightarrow v_{21} = -1/2$$

$$v_1 = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$$

$\rightarrow \quad (A - \lambda I)\, v_2 = v_1$

$$\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} v_{22} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$$

$$2v_{12} + 4v_{22} = 1$$

$$v_{12} = 1/2 \quad v_{22} = 0$$

$$Q = \begin{bmatrix} 1 & 1/2 \\ -1/2 & 0 \end{bmatrix} \rightarrow \left[Q = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \right]$$

Questão 9: $\dot{\bar{v}} = \bar{A}\bar{v} \quad \bar{v}(0) = \bar{v}_0 \quad y = \bar{c}\bar{v}$

$$y(t) = 5t^2 \cos(2t)$$

$$\lambda = \pm 2j \longrightarrow (\lambda^2+4)^3 \Rightarrow (\sigma + j\beta)$$

mult. 3

$$\bar{A} = \begin{bmatrix} \sigma & -\beta & 1 & 0 & 0 & 0 \\ \beta & \sigma & 0 & 1 & 0 & 0 \\ 0 & 0 & \sigma & -\beta & 1 & 0 \\ 0 & 0 & \beta & \sigma & 0 & 1 \\ 0 & 0 & 0 & 0 & \sigma & -\beta \\ 0 & 0 & 0 & 0 & \beta & \sigma \end{bmatrix} = \begin{bmatrix} 0 & -2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$e^{\bar{A}t} = \begin{bmatrix} \cos 2t & -\sin 2t & t\cos 2t & -t\sin 2t & \frac{t^2}{2}\cos 2t & -\frac{t^2}{2}\sin 2t \\ \sin 2t & \cos 2t & t\sin 2t & t\cos 2t & \frac{t^2}{2}\sin 2t & \frac{t^2}{2}\cos 2t \\ \emptyset & & \cos 2t & -\sin 2t & t\cos 2t & -t\sin 2t \\ & & \sin 2t & \cos 2t & t\sin 2t & t\cos 2t \\ \emptyset & & \emptyset & & \cos 2t & -\sin 2t \\ & & & & \sin 2t & \cos 2t \end{bmatrix}$$

$$y = \bar{c}\bar{v} \qquad \bar{v} = e^{\bar{A}t}\bar{v}_0$$

$$\bar{v}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$5t^2\cos(2t) = \bar{c} \cdot \begin{bmatrix} \frac{t^2}{2}\cos 2t \\ \frac{t^2}{2}\sin 2t \\ t\cos 2t \\ t\sin 2t \\ \cos 2t \\ \sin 2t \end{bmatrix} \rightarrow \bar{c} = [10\ 0\ 0\ 0\ 0\ 0]$$

Questão 10: $h(t) = ?$ $\quad v(0) = 0$

$$\dot{v} = \underbrace{\begin{bmatrix} -7 & -10 \\ 1 & 0 \end{bmatrix}}_{A} v + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{b} x$$

$$y = \underbrace{\begin{bmatrix} 1 & 17 \end{bmatrix}}_{c} v$$

$$H(s) = 1$$

$$Y(s) = c\,(sI - A)^{-1}\, b\, X(s)$$

$$Y(s) = \begin{bmatrix} 1 & 17 \end{bmatrix} \underbrace{\begin{bmatrix} s+7 & 10 \\ -1 & s \end{bmatrix}^{-1}}_{\frac{1}{s^2+7s+10}\begin{bmatrix} s & -10 \\ 1 & s+7 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot 1$$

$$Y_H(s) = \frac{1}{s^2+7s+10} \cdot \begin{bmatrix} 1 & 17 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{s+17}{s^2+7s+10} = \frac{s+17}{(s+2)(s+5)}$$

$$= \frac{A}{(s+2)} + \frac{B}{(s+5)} \qquad A = 5, \quad B = -4$$

Transformada

$\mathcal{L}\{e^{at}\mu(t)\} = \frac{1}{s-a}$

$$\boxed{y_h(t) = \left(5e^{-2t} - 4e^{-5t}\right)\mu(t)}$$