Fourier

$$G_T(t) = u(t+T/2) - u(t-T/2)\ , \quad \text{Tri}_{2T}(t) = \frac{1}{T} G_T(t) * G_T(t)\ , \quad x(t) * y(t) = \int_{-\infty}^{+\infty} x(\beta) y(t-\beta) d\beta$$

$$X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t) \exp(-j\omega t) dt \ \ , \qquad x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \exp(j\omega t) d\omega$$

$$\int_{-\infty}^{+\infty} \mid x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mid X(\omega)|^2 d\omega \ \ , \qquad \mathcal{F}\{x(t)\} = X(\omega) \ \Leftrightarrow \ \mathcal{F}\{X(t)\} = 2\pi x(-\omega)$$

$$\mathcal{F}\{G_T(t)\} = T\text{Sa}(\omega T/2),\ \text{Sa}(x) = \frac{\text{sen}(x)}{x},\ \mathcal{F}\{\text{Sa}(\omega_0 t/2)\} = \frac{2\pi}{\omega_0} G_{\omega_0}(\omega),\ \mathcal{F}\{\text{Sa}^2(\omega_0 t/2)\} = \frac{2\pi}{\omega_0} \text{Tri}_{2\omega_0}(\omega)$$

$$\mathcal{F}\{\delta(t)\} = 1,\ \mathcal{F}\{1\} = 2\pi\delta(\omega),\ \mathcal{F}\{u(t)\} = \pi\delta(\omega) + \frac{1}{j\omega},\ \mathcal{F}\Big\{\mathcal{I}_x(t) = \int_{-\infty}^{t} x(\beta) d\beta\Big\} = X(\omega)\Big(\pi\delta(\omega) + \frac{1}{j\omega}\Big)$$

$$\mathcal{F}\{\exp(-a|t|)\} = \frac{2a}{a^2+\omega^2},\ a>0,\ \mathcal{F}\{\text{sinal}(t)\} = \frac{2}{j\omega},\ \mathcal{F}\{x(t-\tau)\} = X(\omega)\exp(-j\omega\tau),\ \mathcal{F}\{x(-t)\} = X(-\omega)$$

$$\mathcal{F}\{\delta(t-\tau)\} = \exp(-j\omega\tau),\ \mathcal{F}\{x(t)\exp(j\omega_0 t)\} = X(\omega-\omega_0),\ \mathcal{F}\big\{x(t) * y(t)\big\} = X(\omega)Y(\omega)$$

$$\mathcal{F}\{\cos(\omega_0 t)\} = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0), \quad \mathcal{F}\{\text{sen}(\omega_0 t)\} = \frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$$

$$\mathcal{F}\{\sum_{k=-\infty}^{+\infty} \delta(t-kT)\} = \omega_0 \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_0), \quad \omega_0 = \frac{2\pi}{T}$$

$$\mathcal{F}\Big\{\frac{d}{dt}x(t)\Big\} = (j\omega)X(\omega)\ , \qquad \mathcal{F}\{x(t)y(t)\} = \frac{1}{2\pi} X(\omega) * Y(\omega) \quad , \quad \mathcal{F}\{t^m x(t)\} = j^m \frac{d^m}{d\omega^m} X(\omega)$$

Laplace

$$H(s) = \mathcal{L}\{h(t)\} = \int_{-\infty}^{+\infty} h(t)\exp(-st) dt \ \ , \ \ s \in \Omega_h \quad , \quad \int_{-\infty}^{+\infty} x(t) dt = X(s)\Big|_{s=0,\ \ 0\in\Omega_x}$$

$$\mathcal{L}\{\delta(t)\} = 1,\ s \in \mathbb{C} \ \ , \ \ \mathcal{L}\{x(t) = x_1(t) * x_2(t)\} = \mathcal{L}\{x_1(t)\}\mathcal{L}\{x_2(t)\} \quad , \ \ \Omega_x = \Omega_{x_1} \cap \Omega_{x_2}$$

$$\mathcal{L}\{y(t) = x(t-\tau)\} = X(s)\exp(-s\tau) \quad , \ \ \Omega_y = \Omega_x \quad , \ \ \mathcal{L}\{\exp(-at)u(t)\} = \frac{1}{s+a} \quad , \quad \text{Re}(s+a) > 0$$

$$\mathcal{L}\{\exp(-\alpha t)\cos(\beta t)u(t)\} = \frac{(s+\alpha)}{(s+\alpha)^2+\beta^2}\ , \mathcal{L}\{\exp(-\alpha t)\text{sen}(\beta t)u(t)\} = \frac{\beta}{(s+\alpha)^2+\beta^2}\ , \ \text{Re}(s+\alpha) > 0$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}\exp(-at)u(t)\Big\} = \frac{1}{(s+a)^{m+1}}\ , \ \ \text{Re}(s+a) > 0\ , \ m \in \mathbb{N}$$

$$\mathcal{L}\Big\{y(t) = \int_{-\infty}^{t} x(\beta)u(\beta) d\beta\Big\} = \frac{1}{s}\mathcal{L}\{x(t)\} \ \ , \ \ \Omega_y \supset \Omega_x \cap \{s \in \mathbb{C} : \text{Re}(s) > 0\}$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}u(t)\Big\} = \frac{1}{s^{m+1}}\ , \ \ \text{Re}(s) > 0\ , \ m \in \mathbb{N} \quad , \quad \mathcal{L}\{x(-t)\} = X(-s) \quad , \quad -s \in \Omega_x$$

$$\mathcal{L}\{y(t) = \exp(-at)x(t)\} = X(s+a) \quad ; \quad \Omega_y = (s+a) \in \Omega_x$$

$$\mathcal{L}\{y(t) = t^m x(t)\} = (-1)^m \frac{d^m X(s)}{ds^m} \quad , \ \ \Omega_y = \Omega_x\ , \ m \in \mathbb{N} \quad , \quad \mathcal{L}\{\dot{x}(t)\} = sX(s) \quad , \ \ \Omega_{\dot{x}} \supset \Omega_x$$

Laplace (unilateral)

$$\mathcal{L}\{x(t)\} = \int_0^{+\infty} x(t)\exp(-st)dt, \quad \mathcal{L}\{\dot{x}(t)\} = s\mathcal{L}\{x(t)\} - x(0) \ \ , \ \ s \in \Omega_x$$

$$\mathcal{L}\left\{x^{(m)}(t) = \frac{d^m x(t)}{dt^m}\right\} = s^m\mathcal{L}\{x(t)\} - \sum_{k=0}^{m-1} s^{m-k-1}x^{(k)}(0)$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}\exp(-at)u(t)\Big\} = \frac{1}{(s+a)^{m+1}} \ , \ \ \mathrm{Re}(s+a) > 0 \ , \ m \in \mathbb{N}$$

$$\mathcal{L}\{\cos(\beta t)\exp(-at)u(t)\} = \frac{s+a}{(s+a)^2+\beta^2} \ \ , \ \mathrm{Re}(s+a) > 0$$

$$\mathcal{L}\{\mathrm{sen}(\beta t)\exp(-at)u(t)\} = \frac{\beta}{(s+a)^2+\beta^2} \ \ , \ \mathrm{Re}(s+a) > 0$$

$$x(0^+) = \lim_{t\to 0^+} x(t) = \lim_{s\to +\infty} sX(s) \ , \quad \lim_{t\to +\infty} x(t) = \lim_{s\to 0} sX(s)$$

Coeficientes a determinar (equações diferenciais)

$$D(p)y(t) = 0 \quad \Rightarrow \quad y(t) = \sum_{k=1}^{m} a_k f_k(t), \qquad f_k(t) \text{ modos próprios (considerando multiplicidades)}$$

Se λ é raiz de multiplicidade r de $D(\lambda)$, então $\exp(\lambda t)$, $t\exp(\lambda t)$, $\ldots$, $t^{r-1}\exp(\lambda t)$ são modos próprios.

$$D(p)y(t) = N(p)x(t) \ \ , \ \text{se} \ \ \bar{D}(p)x(t) = 0 \text{ então } \ \bar{D}(p)D(p)y(t) = 0$$

$$\text{Solução forçada:} \quad y(t) = y_h(t) + y_f(t) \quad \Rightarrow \quad D(p)y_f(t) = N(p)x(t) \ , \ D(p)y_h(t) = 0$$

$$y_f(t) = \sum_{k=1}^{m} b_k g_k(t), \qquad g_k(t) \text{ modos forçados (considerando multiplicidades e ressonâncias)}$$

Variáveis de estado: $\dot{v}(t) = f(v(t), x(t), t)$, $y(t) = g(v(t), x(t), t)$

Pontos de equilíbrio: $\bar{v}$ tais que $f(\bar{v}, \bar{x}) = 0$, $\bar{x}$ =cte.

Sistema linear (em torno dos pontos de equilíbrio)

$$A = \left[\frac{\partial f_i}{\partial v_j}\right]\Bigg|_{\bar{v},\bar{x}} \ , \ B = \left[\frac{\partial f_i}{\partial x_j}\right]\Bigg|_{\bar{v},\bar{x}} \ , \ C = \left[\frac{\partial g_i}{\partial v_j}\right]\Bigg|_{\bar{v},\bar{x}} \ , \ D = \left[\frac{\partial g_i}{\partial x_j}\right]\Bigg|_{\bar{v},\bar{x}}$$

$$\frac{N(p)}{D(p)} = \frac{\beta_2 p^2 + \beta_1 p + \beta_0}{p^3 + \alpha_2 p^2 + \alpha_1 p + \alpha_0} + \beta_3, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \ c = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 \end{bmatrix}, d = \begin{bmatrix} \beta_3 \end{bmatrix}$$

$$\dot{v} = Av + bx \ , \ y = cv + dx \ , \ \frac{N(p)}{D(p)} = c(p\mathrm{I} - A)^{-1}b + d = b'(p\mathrm{I} - A')^{-1}c' + d$$

$$v = T\hat{v} \quad \Rightarrow \quad \hat{A} = T^{-1}AT, \ \hat{b} = T^{-1}b, \hat{c} = cT \ \ , \ \ T \text{ não singular}$$

A representação entrada-saída é invariante com transformações de similaridade.